

STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI – 600 086

M. Phil. DEGREE EXAMINATION FEBRUARY 2018

DEPARTMENT OF MATHEMATICS

ADVANCED STRUCTURES IN ALGEBRA

CLASS: M.PHIL.

MAX: 100 MARKS

CODE: 16MT/RO/SA1 05

TIME: 3 HOURS

Section A

Answer any Five questions (5 × 8 = 40)

1. Define semisimple ring. Prove that every R -module of a semisimple ring is semisimple.
2. Define Projective module and free module. Prove that every free module is projective.
3. State and prove Schanuel's lemma.
4. Define Injective module and give an example.
5. Prove that the homomorphic image of a semisimple algebra is semisimple.
6. Prove that every projective module of rank n over a semi-local ring is free of rank n .
7. Define a semiring. Give two examples.

Section B

Answer any Three questions (3 × 20 = 60)

8. State and prove Jacobson's density theorem on semi simple R - modules.
 9. State and prove Wedderburn's Structure theorem on semi-simple R – algebras
 10. State and prove the equivalent conditions for a semisimple algebra to be simple
 11. a) State and prove Baer's Theorem.
b) Prove that every Z -module can be embedded in an injective Z -module.
 12. a) State and prove Schanuel's lemma.
b) State and prove the equivalent conditions for a module to be projective.
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