STELLA MARIS COLLEGE(AUTONOMOUS), CHENNAI - 86.
(For candidates admitted during the academic year 2017-18)

SUBJECT CODE: 16MT/RO/GT1 05

M. Phil DEGREE EXAMINATION, FEBRUARY 2018
BRANCH I – MATHEMATICS
FIRST SEMESTER

CLASS : M.Phil
PAPER: GRAPH THEORY
TIME : 3 HOURS **MAX. MARKS: 100**

SECTION - A

ANSWER ANY FIVE: (5 x 8 = 40)

1. State and prove Dirac's condition for Hamiltonicity of a graph.
2. Define a degree sequence and prove that the non-negative integers $d_1, d_2, \ldots, d_n$ are the vertex degrees of some graph if and only if $\sum_i d_i$ is even.
3. Prove that a graph G has a 1-factor if and only if $o(G-S) \leq |S|, \forall S \subseteq V(G)$.
4. Define matching and prove Berge's condition for matching.
5. Prove that edges in a plane graph G form a cycle in G, if and only if the corresponding dual edges form a bond in G*.
6. Explain the characteristics of interval graphs with an example.
7. Draw a Circulant graph $C(15, \pm\{1,2\})$ and explain the properties of a Circulant graph.

SECTION - B

ANSWER ANY THREE: (3 x 20 = 60)

8. a) Define centre of a graph and prove that centre of a tree is a vertex or an edge.

   b) Prove that for an n-vertex graph G (with $n \geq 1$) the following are equivalent:

   i) G is connected and has no cycles.

   ii) G is connected and has $n-1$ edges.

   iii) G has n -1 edges and has no cycles.

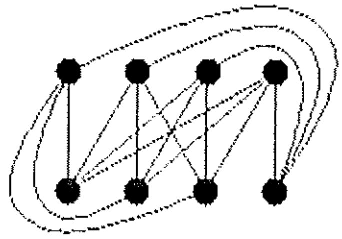
   iv) G has no loops and has for each $u, v \in V(G)$, exactly one u-v path. (8+12)

9. a) If G is a 3-regular graph prove that $\kappa(G) = \kappa'(G)$.

   b) State and prove Hall's theorem. (8+12)

10. a) State and prove for Kuratowski's theorem.

b) Define chromatic number and crossing number and find the same for the following graph.



(12 +8)

11. a) Define circular graphs and interval graphs with examples.

b) Prove that if G is a chordal graph then every minimal vertex cut of G induces a clique in G. (8+12)

12. a) Explain Interconnection networks and give any 10 properties.

b) Explain domination problem and find the same for an Hypercube of dimension 3.

(12+8)