STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI –86

(For candidates admitted from 2016 onwards)

M.PHIL. Degree Examination, February 2018

Paper: FUNCTIONAL ANALYSIS
Time: 3hrs

Code:16MT/RO/ FA 105
Max: 100marks

SECTION A (5x8=40)
Answer any FIVE questions

1. .Discuss the seperability of $L_p(0,1)$

2. State and prove Banach contraction principle.

3. Prove that if X is a commutative Banach algebra with identity e , then $\|e - x\| < 1$ implies that x has an inverse.

4. Show that a bounded linear operator $P : H \to H$ on a Hilbert space H is a projection if and only if P is self adjoint and idempotent

5.Prove that the sum and product of projections is a projection.

6..Prove that if the Frechet differential exists then the weak differential also exists and $df(x,h) = Df(x,h)$

7. Suppose X and Y are normed spaces then prove that to each $T \in B(X,Y)$ there corresponds a unique $T^* \in B(Y^*, X^*)$ that satisfies $< Tx, y^* =< x, T^* y^* >$ for all $x \in X$ and $y^* \in Y^*$.

SECTION B (3x20=60)
Answer any THREE questions

8.a) State and prove Hausdorff theorem.

b) Prove that a complete metric space is of category II.

9. a) Prove that if X is a Banach algebra x^{-1} exists if and only if x is in no ideal.

b)If $x \in X$ is never 0 prove that its inverse never exist.

10. Let $T:X\to X$ be a compact linear operatoron a normed space X.

Prove that for every $\lambda \neq 0$ the range of $T_\lambda = T - \lambda I$ is closed.

11. If an nth partial derivative of the function f exists in a neighbourhood of the point $T_0 = (t_1^{(0)}, t_2^{(0)} ...... t_n^{(0)})$ and if this derivative is continuous at T_0 then prove that the nth partial difference - derivative also exists at T_0 . Further prove that both the derivatives coincide

12. State and prove Haar Uniqueness theorem.

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