STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2016 – 17)

SUBJECT CODE : 16MT/RC/TG105

M.Phil. DEGREE EXAMINATION, FEBRUARY 2017
MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : ADVANCED TOPOLOGY AND GEOMETRY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ANY FIVE QUESTIONS: **(5 x 8 = 40)**

1. Prove that a space X is contractible if and only if X is of the same homotopy type as a single point.
2. If $(\tilde{X}, p)$ is a covering space of $X, \tilde{x} \in \tilde{X}$, then show that $p_* : \pi_1(\tilde{X}, \tilde{x}) \to \pi_1(X, p(\tilde{x}))$ is injective.
3. Define simplicial complex. If K is simplicial complex of dimension m, then prove that mesh $K^{(1)} \leq (m/(m+1)$ mesh K.
4. If ω is a smooth 1-form, V and W are smooth vector fields on X, show that
$$d\omega(V,W) = \frac{1}{2}\{V(\omega(W)) - W(\omega(V)) - \omega([V,W])\}.$$
5. Prove that the maps $C_{l-1}(K, \mathcal{G}) \overset{\partial}{\leftarrow} C_l(K, \mathcal{G}) \overset{\partial}{\leftarrow} C_{l+1}(K, \mathcal{G})$ satisfy $\partial^2 = \partial \circ \partial = 0$.
6. Let φ be a simplicial approximation to $f : [K] \to [L]$.Let K_1 be a subcomplex of K,and suppose that the restriction of f to $[K_1]$ is simplicial map. Then show that there exists a homotopy between f and φ which is stationary on $[K_1]$.
7. If X and Y are smooth and $\Psi : X \to Y$ is a smooth map, prove that $d \circ \Psi^* = \Psi^* \circ d$.

SECTION B

ANSWER ANY THREE QUESTIONS: (3 x 20 = 60)

8. (a) Define homotopy and show that it is an equivalence relation.

   (b) If X, Y, Z be arcwise connected and $x_0 \in X$, show that

   (i) if $f: X \to Y$ and $g: Y \to Z$ are continuous, then $(g \circ f)_* = g_* \circ f_*$.

   (ii) if f_0 and $f_1: X \to Y$ are homotopic maps and $F: X \times I \to Y$ is a homotopy from f_0 to f_1 ,then $(f_1)_* = \sigma_\# \circ (f_0)_*$, where σ is a path in Y from $f_0(x_0)$ to $f_1(x_0)$ given by $\sigma(t) = F(x_0, t)$.

9. Define covering space. State and prove Covering homotopy theorem.

10. (a) If $[s] = [v_0, v_1, \dots v_k]$ is a k-simplex and $v \in (s)$, prove that $(v, [s^{k-1}])$ is in general position and $v * [s^{k-1}] = [s]$.

    (b) If s is a simplex in R^n , then show that diam$[s] = \rho(v_1, v_2)$ for some pair v_1, v_2 of vertices of s.

11. State and prove all the properties of exterior differentiation operator d.

12. (a) State and prove Poincare's lemma.

    (b) State and prove Inverse function theorem.