STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2016–17)

SUBJECT CODE : 16MT/RC/AA105

M.Phil. DEGREE EXAMINATION, FEBRUARY 2017
MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : **ADVANCED ALGEBRA AND ANALYSIS**
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ANY FIVE QUESTIONS: **(5 x 8= 40)**

1. (a) Distinguish between a partially ordered set and a totally ordered set.
(b) Prove that any totally ordered set is distributive Lattice.
2. If a and b are elements of a Modular lattice, then prove that the map $x \rightarrow x \Lambda b$ is an isomorphism of the interval $I[\,a,\ a\, v\ b\,]$ onto $I[\,a\, \Lambda\, b, b\,]$ and that the inverse isomorphism is $y \rightarrow y\, v\ a$.
3. If J is a nil left ideal in an Artinian ring R, then prove that J is Nilpotent.
4. Let R be a Noetherian ring having no non-zero Nilpotent ideals. Then prove that R has no nonzero nil ideals.
5. (a) Define a tensor product of a right R-module and a left R-module.
(b) Let K be a commutative ring and let A and B be K-algebras. Define the tensor product of A and B.
(c) When do you call a topological space, an Hausdorffspace ?
6. In a topological vector space X, prove that
(a) every neighbourhood of 0 contains a balanced neighbourhood of 0, and
(b) every convexneighbourhood of 0 contains a balanced convex neighbourhood of 0.
7. State and prove the Jensen's inequality on the positive measure on a σ – algebra.

SECTION B

ANSWER ANY THREE QUESTIONS: **(3 x 20= 60)**

8. State and prove the Fundamental theorem of Projective Geometry.
9. State and prove the Wedderburn – Artin theorem.
10. State and prove the Urysohn's lemma.

11. If X is topological vector space with a countable local base, then prove that there is a metric d on X satisfying the following :

(a) d is compatible with the topology of x

(b) the open balls centered at 0 are balanced, and

(c) d is invariant : $d\ (x + z\ , y + z) = d\ (x\ , y)$ for x, y, z € X. If, in addition, X is locally convex, then d can be chosen so as to satisfy (a), (b), (c) and also (d) all open balls are convex.

12. State and prove the Plancherel theorem on Fourier transforms.