STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16 & thereafter)

SUBJECT CODE : 15MT/PC/TO34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2018
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : CORE
PAPER : TOPOLOGY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL THE QUESTIONS: **(5 ×2 = 10)**

1. Show that finite union of closed sets is closed.
2. Define a linear continuum and give an example.
3. State the finite intersection property.
4. State Tietz extension theorem.
5. Define the box topology.

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5 × 6 = 30)**

6. Define a Hausdorff space and show that every finite point set in a Hausdorff space X is closed.
7. State and prove the intermediate value theorem.
8. Show that every compact subspace of a Hausdorff space is closed.
9. Prove that every metrizable space is normal.
10. Let $f: A \to \prod_{\alpha \in J} X_\alpha$ defined by $f(a) = \left(f_\alpha(a)\right)_{\alpha \in J}$ where $f_\alpha: A \to X_\alpha$ for each α. Let $\prod X_\alpha$ have the product topology. Show that f is continuous if and only if each function f_α is continuous.
11. Show that the union of a collection of connected subspaces of X that have a point in common is connected.
12. Show that every compact space is limit point compact.

SECTION – C

ANSWER ANY THREE QUESTIONS: (3 ×20 = 60)

13. (a) Let $\mathcal{B}$ and $\mathcal{B}'$ be bases for the topologies τ and τ' respectively on X. Prove that the following conditions are equivalent.

    (i) τ' is finer than τ.

    (ii) For each $x \in X$, and each basis element $B \in \mathcal{B}$ containing x, there is a basis element $B' \in \mathcal{B}'$ such that $x \in B' \subset B$.

    (b) If A is a subspace of X and B is a subspace of Y, then show that the product topology on $A \times B$ is the same as the topology $A \times B$ inherits as a subspace of $X \times Y$.

    (c) Let Y be a subspace of X. Show that a set A is closed in Y is and only if it is equals the intersection of a closed set of $X$with Y.

14. (a) If L is a linear continuum in the order topology, then show that L is connected.

    (b) Define locally connected space and show that a subspace X is locally connected if and only if for every open set U of X, each component of U is open in X.

15. (a) Show that product of finitely many compact spaces is compact.

    (b) State and prove uniform continuity theorem.

16. State and prove Urysohn's metrization theorem.

17. State and prove the Tychonoff theorem.