STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16 & thereafter)

SUBJECT CODE : 15MT/PC/RA14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2018
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE	**:**	**CORE**
PAPER	**:**	**REAL ANALYSIS**
TIME	**:**	**3 HOURS**

MAX. MARKS : 100

SECTION – A **(5 X 2 = 10)**
ANSWER ALL QUESTIONS

1. Distinguish between adherent points and accumulation points of $\mathbb{R}^n$.
2. Determine the Cesaro sum of the series $\sum_{n=0}^{\infty} z^n$, $|z| = 1, z \neq 1$.
3. Consider the sequence of functions $f_n(x) = n^2 x(1-x)^n,\ 0 \leq x \leq 1,\ n = 1,2,...$Is it true that $\lim_{n\to\infty} \int_0^1 f_n(x)\, dx = \int_0^1 \lim_{n\to\infty} f_n(x)\, dx$? Justify your claim.
4. State mean value theorem for functions $f: S \to \mathbb{R}^m, S \subset \mathbb{R}^n$.
5. If $f: S \to \mathbb{C}$, $S \subset \mathbb{C}$, represented by $f(z) = u + iv$ is dfferentiable determine the Jacobian determinant of f.

SECTION – B **(5 X 6 = 30)**
ANSWER ANY FIVE QUESTIONS

6. State and prove Heine-Borel theorem.
7. State and prove Lindelof covering theorem.
8. Prove that the infinite product $\prod_{n=0}^{\infty}(1 + z^{2^n})$ converges if $|z| < 1$.
9. Prove that every convergent series is Cesarosummable.
10. State and prove Weierstrass M-test.
11. Let $f(x,y) = \begin{cases} \frac{xy(x^2-y^2)}{(x^2+y^2)} & if\ (x,y) \neq (0,0) \\ 0 & if\ (x,y) = (0,0) \end{cases}$. Check the equality of the mixed partial derivatives $D_{1,2}f$ and $D_{2,1}f$ at (0,0).
12. Assume that $f = (f_1, f_2, \ldots, f_n)$ has continuous partial derivatives $D_j f_i$ on an open set S in $\mathbb{R}^n$, and that $J_f(a) \neq 0$ for some point a in S. Then prove that there is an n-ball $B(a)$ on which f is one-to-one.

SECTION – C **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

13. State and prove Bolzano-Weierstrass theorem.

14. (a) State and prove Mertens theorem.

 (b) If $a_n > 0$ then prove that the product $\prod(1 + a_n)$ converges if and only if, the series $\sum a_n$ converges.

15. (a) State and prove Cauchy condition for uniform convergence for a sequence of functions defined on a set S.

 (b) State and prove Bernstein,s theorem.

16. (a) If one of the partial derivatives of a function f at a point c exists and the remaining n-1 partial derivatives exists in some n-ball $B(c)$ and are continuous at c then prove that f is differentiable at c.

 (b) State and prove Taylor's formula.

17. State and prove implicit function theorem.

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