STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16& thereafter)

SUBJECT CODE : 15MT/PC/MA14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2018
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE	**:**	**CORE**
PAPER	**:**	**MODERN ALGEBRA**
TIME	**:**	**3 HOURS** **MAX. MARKS: 100**

SECTION – A

ANSWER ALL THE QUESTIONS: (5 ×2 = 10)

1. If G is a group of order 25, then what can you say about the centre of the group G? Give reasons for your answer.
2. State Fermat's theorem.
3. Is there an irreducible polynomial of degree 3 over the field of real numbers? If yes, give an example of such a polynomial or if no, give reasons.
4. Define an algebraic number and give an example.
5. Define the Galois group of a polynomial $f(x) \in F[x]$.

SECTION – B

ANSWER ANY FIVE QUESTIONS: (5 × 6 = 30)

6. Suppose that G is the internal direct product of $N_1, N_2, \cdots, N_n$. Then prove that for $i \neq j$ $N_i \cap N_j = (e)$, and if $a \in N_i, b \in N_j$ then $ab = ba$.
7. Let R be a Euclidean ring and a, b *in* R. If $b \neq 0$ is not a unit in R, then prove that $d(a) < d(ab)$.
8. State and prove the Eisenstein Criterion about the irreducibility of a polynomials over the field of rational numbers.
9. If a rational number r is also an algebraic integer, then prove that r must be an ordinary integer.
10. Prove that a group G is solvable if and only if $G^{(k)} = (e)$ for some k.
11. If G is a group of order 108 then show that G has a normal subgroup of order 9 or 27.
12. Find the splitting field of the polynomial $x^3 - 2 \in Q[x]$ and also find its degree over the field Q of rational numbers.

SECTION – C

ANSWER ANY THREE QUESTIONS: **(3 × 20 = 60)**

13. (a) Prove that any two p-Sylow subgroups of a finite group G are conjugates.
    (b) State and prove Cauchy's theorem of a finite group G.

14. (a) Prove that the ring of Gaussian integers is a Euclidean ring.
    (b) Prove that any prime number of the form $4n + 1$can be expressed as a sum of two squares.

15. (a) State and prove the division algorithm in the ring of polynomials $F[x]$ over a field F.
    (b) Prove that a product of two primitive polynomials over the rational field is a primitive polynomial.

16. Prove that an element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F.

17. If F is a field which contain all the nth roots of unity for every integer n and if $p(x)$ in $F[x]$ is solvable by radicals over F , then prove that the Galois group over F of $p(x)$ is a solvable group.