STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16 and thereafter)

SUBJECT CODE : 15MT/PC/FD34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2018
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : CORE
PAPER : FLUID DYNAMICS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL THE QUESTIONS: **(5 x 2 = 10)**

1. Define stream lines.
2. Define Beltrami vector.
3. Give examples for axi – symmetric flows.
4. Define line sink.
5. State the use of components of stress.

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5 x 6 = 30)**

6. At the point in an incompressible fluid having spherical polar coordinates (r, θ, ψ), the velocity components are $[2Mr^{-3}cos\theta,\ Mr^{-3}sin\theta,\ 0]$,where M is a constant.Show that the velocity is of the potential kind.Find the velocity potential and the equations of the streamlines.
7. Obtain the acceleration of a fluid.
8. Derive Bernoulli's equation of motion.
9. Obtain doublet in a uniform stream.
10. Discuss the flow for which $w = z^2$.
11. Show how the circle theorem applied to determine modified flows when a long circular cylinder is introduced into a given 2 dimensional flow.
12. State and prove Uniqueness theorem.

SECTION – C

ANSWER ANY THREE QUESTIONS: **(3 x 20 = 60)**

13. a) Show that at all points of the field of flow the equipotentials are cut orthogonally by the streamlines.

b) Liquid flows through a pipe whose surface of revolution of the curve $y = a + kx^2/a$about the x – axis $(-a \le x \le a)$. If the liquid enters at the end $x = -a$of the pipe with velocity V,show that the time taken by a liquid particle to traverse the entire length of the pipe from $x = -a$ to $x = +a$ is $\{2a/V(1+k)^2\}\,(1 + \frac{2}{3}k + \frac{1}{5}k^2)$. (5+15)

14. a)AB is a tube of small uniform bore forming a quadrantal arc of a circle of radius a andcentre O,OA being horizontal and OB vertical with B below O.The tube is full of liquid of density ρ ,the end B being closed .If B is suddenly opened, show that initially $/dt = 2g/\,\pi$,where $u = u(t)$ is the velocity, and that the pressure at a pointwhose angular distance from A is θ immediately drops to ρga ($\sin\theta - \frac{2\theta}{\pi}$) above atmospheric pressure.Prove further that when the liquid remaining in the tube subtends an angle β at the centre,$\frac{d^2\beta}{d\,t^2} = -\frac{2g}{a\beta} sin^2\left(\frac{\beta}{2}\right)$.

    b) Discuss the case of steady motion under conservative body forces.

15. (a) Describe doublet and find the velocity potential at a point P due to a doublet at O.

    (b)Doublets of strengths μ_1, μ_2 are situated at points A_1, A_2whose Cartesian coordinates are $(0,0,c_1), (0,0,c_2)$, their axes being directed towards and away from the origin respectively.Find the condition that there is no transport of fluid over the surface of the sphere $x^2 + y^2 + z^2 = c_1c_2$.

16. a) Discuss the two dimensional flow for which$w = (ik/2\pi)\ log\ z$.

    b) Describe the irrotational motion of an incompressible liquid for which the complex potential is $w = ik\ logz$. Two parallel line vortices of strengths $k_1,\ k_2(k_1 + k_2\ \neq\ 0)$ in unlimited liquid cross the z-plane at points A, B respectively.The centre of mass of masses k_1 at A and k_2 at $B$is G.Show that if the motion of the liquid is due solely to these vortices,G is a fixed point about whichA, B move in circles with angular velocity $(k_1 + k_2)/(AB)^2$. Show also that the fluid speed at any point P in the z- plane is $(k_1 + k_2)\ CP/(AP.BP)$, where$C$ is the centre of mass of masses $k_2$at A , k_1 at B.

17. a) Discuss the coefficient of viscosity and laminar flow.

    b) Derive Navier – Stokes equation of motion of a viscous fluid.