STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16& thereafter)

SUBJECT CODE : 15MT/PC/DE14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2018
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : DIFFERENTIAL EQUATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

(5 X 2 = 10)

ANSWER ALL QUESTIONS

1. Find the Wronskian of e^{-t}, e^{t} and te^{t} at $t = 0$.
2. Find the second approximation $x_1(t)$ for the initial value problem $x' = x^2$, $x(0) = 1$.
3. Write the auxiliary equations for $p^2 - y^2q = y^2 - x^2$.
4. What is meant by boundary value problem?
5. State Dirichlet problem.

SECTION – B (5 X 6 = 30)

ANSWER ANY FIVE QUESTIONS

6. State and prove Able's formula.
7. Solve the Bessel equation $L(x)(t) = t^2x'' + tx' + (t^2 - p^2)x = 0$, $Re\ p \geq 0$ to get the solution $J_p(t)$, the Bessel function of order p of the first kind.
8. State and prove Gronwall's inequality.
9. Show that if $x(t)$ is a solution of the initial value problem $x' = f(t,x), x(t_0) = x_0$ on I if and only if $x(t)$ is a solution of the integral equation $x(t) = x_0 + \int_{t_0}^{t} f\big(s, x(s)\big)ds$.
10. Solve by Charpit's method $pxy + pq + qy - yz = 0$.
11. Derive the one-dimensional wave equation.
12. Obtain the solution of three dimensional Laplace equation by using the method of separation of variables.

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SECTION – C (3 X 20 = 60)

ANSWER ANY THREE QUESTIONS

13. (a) Solve the Legendre equation $(1 - t^2)\, x'' - 2\, t\, x' + p\,(p+1)\, x = 0$ where p is a real number.

    (b) If P_n is a Legendre polynomial then prove that $\int_{-1}^{1} P_n^2(t)\, dt = \frac{2}{2n+1}$.

14. State and prove Picard's theorem.

15. Reduce the equation $(n\text{-}1)^2 \left(\frac{\partial^2 z}{\partial x^2}\right) - y^{2n} \left(\frac{\partial^2 z}{\partial y^2}\right) = n\, y^{2n-1} \left(\frac{\partial z}{\partial y}\right)$ to canonical and find its general solution.

16. (a) Discuss D'Alembert's solution of wave equation .

    (b) An insulated rod of length *l* has its ends A and B maintained at 0^0 C and 100^0 C respectively until steady state conditions prevail . If B is suddenly reduced to 0^0 C and maintained at 0^0C, find the temperature at a distance *x* from A at time *t*.

17. Solve two dimensional Laplace's equation in plane polar coordinates (r, θ).