STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16 & thereafter)

SUBJECT CODE: 15MT/PC/CA34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2018
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : CORE
PAPER : COMPLEX ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION–A

ANSWER ALL QUESTIONS: **(5×2=10)**

1. Compute $\int_{|z|=1} |z-1||dz|$.
2. When is a cycle γ said to be homologous to zero?
3. State Poisson –Jensen's formula.
4. Define locally bounded family of functions.
5. State Schwarz-Christoffel formula.

SECTION–B

ANSWER ANY FIVE QUESTIONS: **(5×6=30)**

6. State and prove Cauchy's integral formula.
7. If u_1 and u_2 are two harmonic functions in a region Ω then prove that

   $\int_\gamma u_1 * du_2 - u_2 * du_1 = 0$, for every cycle γ which is homologous to zero in Ω.
8. Show that a region Ω in the complex plane is simply connected if and only if $n(\gamma, a) = 0$ for all cycles γ in Ω and all points a which do not belong to Ω.
9. Show that the necessary and sufficient condition for the absolute convergence of the product $\prod_1^\infty (1 + a_n)$ is the convergence of the series $\sum_1^\infty |a_n|$.
10. If $p_1, p_2, . . ., p_n, . . .$ is the ascending sequence of prime numbers and $\sigma = Re\, s > 1$, then prove that $\frac{1}{\zeta(s)} = \prod_{n=1}^\infty (1 - p_n^{-s})$ where $\zeta(s)$ is the Riemann zeta function.
11. Show that the family $\mathfrak{F}$ of functions defined on a fixed region Ω of the complex plane is totally bounded if and only if to every compact set $E \subset \Omega$ and every $\epsilon > 0$ it is possible to find functions $f_1, f_2, . . ., f_n \in \mathfrak{F}$ such that every $f \in \mathfrak{F}$ satisfies $d(f, f_j) < \epsilon$ on E for some f_j.
12. Show that under a conformal mapping a source or sink at a given point corresponds to an equal source or sink at the image of that point.

SECTION–C

ANSWER ANY THREE QUESTIONS: **(3×20 =60)**

13. State and prove Cauchy's theorem for a rectangle.

14. (i) If f is analytic and non-zero in a simply connected region Ω in the complex plane then show that it is possible to define a single valued analytic branch of $log f(z)$ and $\sqrt[n]{f(z)}$ in Ω.

    (ii) Derive Poisson's formula for harmonic functions.

15. (i) Derive the formula $sin \pi z = \pi z \prod_{n=1}^{\infty}\left(1 - \frac{z^2}{n^2}\right)$.

    (ii) Derive the functional equation $\zeta(s) = 2^s \pi^{s-1} sin \frac{\pi s}{2} \Gamma(1-s)\zeta(1-s)$.

16. State and prove Arzela'stheorem.

17. State and prove Riemann mapping theorem.