STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted during the academic year 2015–16 and thereafter)

SUBJECT CODE:15MT/MC/VA34

B. Sc. DEGREE EXAMINATION, NOVEMBER 2017
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE	**:**	**MAJOR – CORE**
PAPER	**:**	**VECTOR ANALYSIS AND APPLICATIONS**
TIME	**:**	**3 HOURS** **MAX. MARKS : 100**

SECTION-A
Answer All the questions **(10 x 2 = 20)**

1. If $\phi\ (x,y,z) = x^2y + y^2x + z^2$ find Grad ϕ at the point (1,1,1).

2. Find the maximum value of the directional derivative of the function $\phi = 2x^2 + 3y^2 + 5z^2$ at the point $(1,1,-4)$.

3. Show that the vector $3x^2y\vec{i} - 4xy^2\vec{j} + 2xyz$ is solenoidal

4. If $\vec{F} = xz3\vec{i} - 2xyz\vec{j} + xz\vec{k}$ find $curl\vec{F}$ at (1,2,0).

5. If $\overrightarrow{f(t)} = (3t^2 - 1)\vec{i} + (2-6t)\vec{j} - 4t\vec{k}$ find $\int_2^3 \overrightarrow{f(t)}dt.$

6. If $\vec{F} = yz\vec{i} + zx\vec{j} - xy\vec{k}$ find $\int_c \vec{t}\,\overrightarrow{dr}$ where c is given by $x = t, y = t^2, z = t^3$ from $P(0,0,0)$ to Q (2,4,8).

7. Find the unit vector normal to the surface $x^2 + 3y^2 + 2z^2 = 6$ at the point (2,0,1)

8. Show that the surface $5x^2 - 2yz - 9x = 0$ and $4x^2y + z^2 - 4 = 0$ are orthogonal at the point (1,-1,2).

9. State Stoke's theorem.

10. State Green's theorem.

SECTION-B
Answer any FIVE questions **(5 x 8 = 40)**

11. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\vec{r}|$ prove that
    a) $\nabla\left\{\frac{1}{r}\right\} = \frac{-\vec{r}}{r^3}$ b) $\nabla\, r^n = nr^{n-2}\vec{r}$

12. Find the angle between the normal to the surface $xy - z^2 = 0$ at the points (1,4,-2) and (-3, -3, -3).

13. Prove that curl of curl $\vec{F} = \nabla(\nabla \cdot \vec{F}) - \nabla^2\vec{F}$

14. The acceleration of a particle at any time $\vec{a} = 12\,cos\,2t\vec{\imath} - 8\,sin\,2t\vec{\jmath} + 16t\vec{k}$. If the velocity $\vec{v}$ and the displacement are zero at time $t = 0$ find the velocity and displacement at any time t.

15. Find $\int_c \vec{F} \cdot d\vec{r}$ where $\vec{F} = z\vec{\imath} + x\vec{\jmath} + y\vec{k}$ along the arc of the curve $\vec{r} = cost\,\vec{\imath} + sin\,t\,\vec{\jmath} + t\,\vec{k}$ from $t = 0$ to $t = 2\pi$.

16. Find the equation of the tangent plane and normal to the surface $xyz = 4$ at the point (1,2,2)

17. Evaluate $\iint(y^2z\vec{\imath} + z^2x\vec{\jmath} + x^2y\vec{k})ds$ where s is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ lying in the positive octant.

18. Using divergence theorem evaluate $\iint \vec{F} \cdot \vec{n}\,ds$ where $\vec{F} = 4xz\vec{\imath} - y^2\vec{\jmath} + yz\vec{k}$ and S is the surface of the cube bounded by the planes $x = 0,\ x = 2,\ y = 0,\ y = 2,$ $z = 0,\ z = 2.$

SECTION-C

Answer any TWO questions (2 x 20 = 40)

19. a) Find ϕ if $\nabla\phi = (y + sinz)\vec{\imath} + x\,\vec{\jmath} + cos\,z\,\vec{k}$.

b) Prove that $\nabla \times (\vec{u} \times \vec{v}) = [(\vec{v}.\nabla)\,\vec{u} - (\nabla.\vec{u})\,\vec{v}] - [(\vec{u}.\nabla)\,\vec{v} - (\nabla.\vec{v})\,\vec{u}]$.

20. a) Find $\int \vec{F} \cdot d\vec{r}$ where $\vec{F} = (x^2 - y^2)\,\vec{\imath} + 2xy\,\vec{\jmath}$ and c is the square bounded by the co-ordinate axes and the lines $x = a, y = a$.

b) If $U = x + y + z, V = x^2 + y^2 + z^2, W = yz + zx + xy$ prove that $(\text{grad}U).(\text{grad}V \times \text{grad}W) = 0$

21. a) Verify Gauss divergence theorem for $\vec{F} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$ taken over the regim bounded by the plane $x = 0, x = a, y = 0, y = a.$

b) Verify Stoke's theorem for $\vec{F} = x^2\vec{\imath} + x\,y\,\vec{\jmath}$ taken round the square in the xy plane whose sides are $x = 0, x = a, y = 0, y = a.$

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