STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16)

SUBJECT CODE : 15MT/MC/RA55

B. Sc. DEGREE EXAMINATION, NOVEMBER 2017
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE : MAJOR – CORE
PAPER : PRINCIPLES OF REAL ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(10X2=20)**
ANSWER ALL THE QUESTIONS

1. When do you say that $f(x)$ approaches 'L' (where $L \epsilon R$) as 'x' approaches 'a'?
2. Prove by an example that if $f(a)$ is not defined, then 'f' cannot be continuous at the point 'a'.
3. State the definition of a Cauchy sequence in a Metric space :
4. Why every subset ofR_d is Open?
5. If A is a bounded subset of a metric space$< M, \rho >$, define the diameter of$A$.
6. Define the "Complete Metric Space".
7. State the Heine-Borel property of a metric space M.
8. Define a Uniformly Continuous function.
9. Define the upper integral of'f' over an interval $[a, b]$.
10. State "The Law of the Mean".

SECTION – B **(5X8=40)**
ANSWER ANY FIVE QUESTIONS

11. The set of all n-tuples of real numbers, denoted by R^n , is a Metric space – Justify.
12. If f and g are real valued functions, if 'f' is continuous at 'a' and if 'g' is continuous at '$f(a)$', then prove that gof is continuous at 'a'.
13. Prove that the arbitrary union of open subsets of a Metric space M is anopen subset of M.
14. Let 'f' be a continuous function from a metric spaceM_1into a Metric space M_2. If the domain of 'f' is connected, prove that the range of'f' is also connected.
15. Let f be a continuous function from the Compact metric space M_1into the Metric space M_2. Using the concept of open covering, prove that the range $f\ (M_1)$ is also compact.
16. Let $'f'$ be a bounded function on $[a, b]$. If σ and τ are any two subdivisions of $[a\ , b]$, then prove that $U[\ f\ ;\ \sigma\ ] \geq L\ [\ f\ ;\ \tau]$.
17. If the real-valued function 'f' has a derivative at the point $c \in R^1$, then prove that 'f' is continuous at 'c'.

SECTION – C **(2X20=40)**
ANSWER ANY TWO QUESTIONS

18. (a) Prove that 'f' is continuous at 'a' , if and only if, $\lim_{n\to\infty} x_n = a$ implies $\lim_{n\to\infty} f(x_n) = f(a)$ (7 + 7)

(b) Let $< M_1, \rho_1 >$and $< M_2, \rho_2 >$ be metric spaces and let $f: M_1 \to M_2$. Prove that'f ' is continuous on M_1 if and only if $f^{-1}(G)$ is open in M_1 , whenever G is open in M_2 (3 + 3)

19. (a) Let $< M, \rho >$ be a metric space. Prove that a subset A of M is totally bounded if and only if every sequence of points of A containsa Cauchy subsequence. (6 + 6)

(b) Let $< M_1, \rho_1 >$be a compact metric space. If f is a continuous function from M_1 into a Metric space $< M_2, \rho_2 >$, then prove that 'f' is uniformly continuous on M_1. (8)

20. (a) If 'f' and 'g' both have derivatives at $c \in R^1$, then prove that $(fg)' = f'(c)\, g(c) + f(c)\, g'(c)$ (5)

(b) State and prove the Fundamental Theorem of Calculus. (15)

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