STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted during the academic year 2015-2016 & thereafter)

SUBJECT CODE : 15MT/MC/GT34

B. Sc. DEGREE EXAMINATION, NOVEMBER 2017
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : MAJOR – CORE
PAPER : INTRODUCTION TO GRAPH THEORY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(10X2=20)**
ANSWER ALL THE QUESTIONS

1. Prove that the sum of the degrees of the points of a graph *G* is twice the number of lines.
2. Let G_1 be a (p_1, q_1) graph and G_2 be a (p_2, q_2) graph then prove that $G_1 + G_2$ is a $(p_1 + p_2, q_1 + q_2 + p_1p_2)$ graph.
3. Show that the partition $P = (7,6,5,4,3,2)$ is not graphic.
4. Define a cycle of length *n*.
5. Define an Eulerian graph and give an example.
6. Prove that every Hamiltonian graph is 2-connected.
7. Define the genus of a graph.
8. Prove that the graph $K_{3,3}$ is not planar.
9. Let G be a (p, q) connected graph, then prove that $q \geq p - 1$.
10. Define dominance matrix of a digraph.

SECTION – B **(5X8=40)**
ANSWER ANY FIVE QUESTIONS

11. (a) Show that in any group of two or more people, there are always two with exactly the same number of friends inside the group.
    (b) Let G be a (p, q) graph then prove that $\Gamma(G) = \Gamma(\bar{G})$.
12. (a) In a graph *G*, prove that any *u*-*v* walk contains a *u*-*v* path.
    (b) If a graph *G* is not connected then prove that the graph $\bar{G}$ is connected.
13. Let *G* be a connected graph with atleast three points then prove that *G* is a block if and only if any two points of *G* lie on a common cycle.
14. (a) If *G* is a graph in which the degree of every vertex is atleast two then prove that *G* contains a cycle.
    (b) Write Fleury's algorithm to construct an eulerian trail in an eulerian graph *G*.

15. If G is a graph with $p \geq 3$ vertices and $\delta \geq {}^{p}/_{2}$, then prove that G is Hamiltonian.

16. State and prove Euler's theorem.

17. (a) Prove that every tree has a centre consisting of either one point or two adjacent points.

(b) In a digraph D, prove that the sum of the indegrees of all the vertices is equal to the sum of their out degrees, each sum being equal to the number of arcs in D.

SECTION – C (2X20=40)

ANSWER ANY TWO QUESTIONS

18. (a) Prove that the maximum number of lines among all p point graphs with no triangles is $\left[\frac{p^2}{4}\right]$.

(b) Let v be a point of a connected graph G. Prove that the following statements are equivalent

(i) v is a cut-point of G.

(ii) There exists a partition of $V - \{v\}$ into subsets U and W such that for each $u \in U$ and $w \in W$, the point v is on every u-w path.

(iii) There exist two points u and w distinct from v such that v is on every u-w path.

(12+8)

19. (a) Prove that a graph G with atleast two points is bipartite if and only if all its cycles are of even length.

(b) If G is a graph then prove that the closure of a graph $c(G)$ is well defined.

(12+8)

20. (a) Let G be a (p, q) graph then prove that the following statements are equivalent

(i) G is a tree.

(ii) Every two points of G are joined by a unique path.

(iii) G is connected and $p = q + 1$.

(iv) G is acyclic and $p = q + 1$.

(b) Define a planar graph and prove that K_5 is non-planar.

(12+8)