

B. Sc. DEGREE EXAMINATION, NOVEMBER 2017  
BRANCH I - MATHEMATICS  
FIRST SEMESTER

COURSE : MAJOR – CORE  
PAPER : DIFFERENTIAL CALCULUS  
TIME : 3 HOURS

MAX. MARKS : 100

SECTION – A (10X2=20)  
ANSWER ALL THE QUESTIONS

1. Write down the nth derivative of  $e^{ax}$ .
2. If  $xy = a e^x + b e^{-x}$ , prove that  $x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} - xy = 0$ .
3. Find the envelope of the family of circles  $a^2 - 2a(x+1) + x^2 + y^2 = 0$ .
4. Find the envelope of the family of straight lines  $y = mx + \frac{a}{m}$ , the parameter being 'm'.
5. Find the radius of curvature for the curve  $xy = 30$  at the point (3,10).
6. Find the coordinates of the centre of curvature of a curve  $y = x^2$  at (1/2,1/4).
7. Define evolute of a curve.
8. Write down the formulae for finding the radius of curvature.
9. State Lagrange's method of undetermined multipliers.
10. Define saddle points.

SECTION – B (5X8=40)  
ANSWER ANY FIVE QUESTIONS

11. Find  $y_n$ , if  $y = \frac{x^2}{(x+2)(x-1)^2}$ .
12. Find the envelope of the family of straight lines  $y + tx = 2at + at^3$ , the parameter being  $t$ .
13. Show that the radius of curvature at any point of the catenary  $y = c \cosh(x/c)$  is equal to the length of the portion of the normal intercepted between the curve and the  $x$ -axis.
14. Show that the evolute of the cycloid  $x = a(\theta - \sin\theta)$  and  $y = a(1 - \cos\theta)$  is another cycloid.
15. Find the radius of curvature of the curve  $r^n = a^n \cos n\theta$ .
16. Find the p-r equation of the cardioids  $r = a(1 - \cos\theta)$ .
17. Trace the curve  $y = (x^2 + 1)/(x^2 - 1)$ .

**SECTION – C**  
**ANSWER ANY TWO QUESTIONS**

(2X20=40)

18. (a) If  $y = \sin ( m \sin^{-1} x )$ , prove that

$$(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} + (m^2 - n^2) y_n = 0.$$

(b) Find the envelope of the circles drawn on the radius vectors of the

ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  as diameter ( 10 + 10 )

19. (a) Find the coordinates of the centre of curvature of the curve  $y^2 = 4ax$  at the point 't'.

(b) Find the chord of curvature through the focus of a parabola and show that it is equal to four times the focal distance of the point. ( 10 + 10 )

20. Discuss the maxima and minima of the function  $x^3 y^2 (6 - x - y)$ .

