STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015–16)

SUBJECT CODE : 15MT/MC/AS55

B. Sc. DEGREE EXAMINATION, NOVEMBER 2017
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE : MAJOR – CORE
PAPER : ALGEBRAIC STRUCTURES
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

Answer all questions: **(10 X 2 =20)**

1. Is $(Z, \cdot)$ a group? If not, why?
2. Define the term 'Groups'.
3. State Fermat's theorem.
4. If G is a finite group and $a \in G$ then Prove that $a^{o(G)} = e$
5. Find the orbits and cycles of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix}$
6. Define 'Inner Automorphism.'
7. State the 'Pigeonhole principle'.
8. When do you say that a ring is a division ring.
9. If U is an ideal of R and $1 \in U$, Prove that U = R.
10. Define maximal ideal.

SECTION – B

Answer any five questions: **(5 X 8 = 40)**

11. State and Prove the necessary and sufficient condition of a subgroup.
12. Show that the cyclic groups are abelian but the converse is false.
13. Prove that a subgroup N of G is a normal subgroup of G if and only if the product of two right cosets of N in G is again a right coset of N in G.
14. If φ is a homomorphism of G into $\bar{G}$ with kernel k then prove that k is a normal subgroup of G.
15. If G is a group then show that A (G), the set of all automorphisms of G, is also a group.
16. Prove that a finite integral domain is a field.
17. If R is a commutative ring with unit element whose only ideals are (o) and R itself prove that R is a field.

SECTION – C

Answer any two questions: (2 x 20 = 40)

18. a) If H and K are finite subgroups of G of orders O(H) and O(K) respectively then prove that $O(HK) = \frac{O(H).O(K)}{O(H \cap K)}$

b) State and prove Lagrange's theorem.

19. a) State and prove the fundamental theorem of homomorphism.

b) Show that every group is isomorphic to a subgroup A(S) for some appropriate S.

20. a) If U is a ideal of the ring R, then prove that R/U is a ring and is a homomorphic image of R.

b) If R is a commutative ring with unit element and M is an ideal of R then prove that M is a maximal ideal of R if and only if R/M is a field.

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