STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2015-16 & thereafter)

SUBJECT CODE : 15MT/PE/FT14

M. Sc. DEGREE EXAMINATION, APRIL 2018
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : ELECTIVE
PAPER : FUZZY SET THEORY AND APPLICATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

Answer all the questions: **5×2=10**

1. For $\alpha \in [0,1]$, define α-cut and strong α-cut of a fuzzy set A in X. Also write the inclusion relationship between them.
2. Define the domain of a fuzzy relation R(X, Y).
3. Prove that every fuzzy complement has at most one equilibrium.
4. Check whether the function given by $A(x) = \begin{cases} \sin x & \text{if } 0 \le x \le \pi \\ 0 & \text{otherwise}. \end{cases}$

   is a fuzzy number.
5. Write any two advantages of fuzzy logic controller

SECTION – B

Answer any five questions: **5×6=30**

6. Order the fuzzy sets defined by the following membership grade functions(assuming $(x \ge 0)$ by the inclusion $(\subset)$ relation:

   $$A(x) = \frac{1}{(1+10x)}, \quad B(x) = \left(\frac{1}{1+10x}\right)^{\frac{1}{2}}, \quad C(x) = \left(\frac{1}{1+10x}\right)^{2}.$$

7. Let $f : X \to Y$ be an arbitrary crisp function. Then for any $A \in \mathcal{F}(X)$, f fuzzified by the extension principle satisfies the equation $f(A) = \bigcup_{\alpha \in [0,1]} f({}_{\alpha+}A)$.
8. State and prove all the properties satisfied by arithmetic operations on closed interval.

9. Let A and B be triangular fuzzy numbers defined as follows:

$$A(x)-\begin{cases}0 & if\ x\leq -1\ and\quad x>3\\ (x+1)/2 & if\quad -1\leq x\leq 1\\ (3-x)/2 & if\quad 1<x\leq 3\end{cases} \quad \text{and} \quad B(x)=\begin{cases}0 & if\quad x\leq 1\quad and\quad x>5\\ (x-1)/2 & if\quad 1<x\leq 3\\ (5-x)/2 & if\quad 3<x\leq 5\end{cases}.$$

Find the sum $A+B$ and the difference $A-B$ of the fuzzy numbers A and B.

10. Explain the methods of designing fuzzy controller.

11. The fuzzy relation R is defined on sets $X_1=\{a,b,c\}$, $X_2=\{s,t\}$, $X_3=\{x,y\}$, $X_4=\{i,j\}$ as follows:

$R(X_1,X_2,X_3,X_4)=\frac{.4}{b},t,y,i+\frac{.6}{a},s,x,i+\frac{.9}{b},s,y,i+\frac{1}{b},s,y,j+\frac{.6}{a},t,y,j+\frac{.2}{c},s,y,i$

compute the projections $R_{1,2,4}, R_{1,3}$ & R_4 also the cylindric extensions $[R_{1,2,4}\uparrow\{X_3\}]$, $[R_{1,3}\uparrow\{X_2,X_4\}]$, $[R_4\uparrow\{X_1,X_2,X_3\}]$.

12. Find the solution of the fuzzy equation $A.X=B$, where A and B are fuzzy numbers on R^+ given as follows.

$$A(x)=\begin{cases}0 & for\ \ x\leq 3 and\ x>5\\ x-3 & for\ 3<x\leq 4\\ 5-x & for\ 4<x\leq 5\end{cases}$$

$$B(x)=\begin{cases}0 & for\ \ x\leq 12 and\ x>32\\ (x-12)/8 & for\ 12<x\leq 20\\ (32-x)/12 & for\ 20<x\leq 32\end{cases}.$$

SECTION – C

Answer any three questions: **3×20=60**

13. (a) For any pair of fuzzy subsets A and B of a universal set X, define the degree of subsethood $S(A,B)$ and prove that $S(A,B)=\frac{|A\cap B|}{|A|}$.

(b) Let $f:X\rightarrow Y$ be a crisp function. Then for any fuzzy subset A of X and for $\alpha\in[0,1]$. Prove that ${}^{\alpha}[f(A)]\supseteq f({}^{\alpha}A)$. Is the reverse side inclusion true? Justify your answer.

(10+10 Marks)

14. Let $f : X \rightarrow Y$ be a crisp function. Then for any $A_i \in F(X)$ and $B_i \in F(Y)$, prove the following properties obtained by the extension principle.

(i) $f(A) = \phi \quad iff \quad A = \phi.$

(ii) If $A_1 \subseteq A_2 \quad then \quad f(A_1) \subseteq f(A_2)$

(iii) If $B_1 \subseteq B_2 \quad then \quad f^{-1}(B_1) \subseteq f^{-1}(B_2)$

(iv) $f(\bigcup_{i \in I} A_i) = \left(\bigcup_{i \in I} f(A_i) \right).$

(iv) $f^{-1}(\bigcup_{i \in I} B_i) = \left(\bigcup_{i \in I} f^{-1}(B_i) \right).$

15. Prove that (a) $u(a, b) = \max(a, b)$ is the only continuous and idempotent fuzzy set union; (b) $i(a, b) = \min(a, b)$ is the only continuous and idempotent fuzzy set intersection.

16. Define the basic operation of addition (+), subtraction (-), multiplication(.) and division(/) of fuzzy numbers A and B. Further, prove that if A and B are continuous fuzzy numbers, prove that A*B is a continuous fuzzy number, where $* \in \{+, -, ., /\}$.

17. Discuss how fuzzy logic control is used in medical industry.