STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2015-16 & thereafter)

SUBJECT CODE : 15MT/PC/LA24

M. Sc. DEGREE EXAMINATION, APRIL 2018
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : CORE
PAPER : LINEAR ALGEBRA
TIME : 3 HOURS **MAX. MARKS : 100**

Section-A
Answer ALL the questions **(5x2=10)**

1. If M is an R-module, then when M is said to be the direct sum.
2. When a linear transform is said to be similar?
3. Define elementary divisors of T in rational canonical form.
4. When do you say the subspace of a vector space is invariant under T?
5. When a matrix $B(nxn)$ is orthogonally equivalent to $A(nxn)$?

Section-B
Answer any FIVE questions **(5x6=30)**

6. Prove that "Any finite abelian group is the direct product (sum) of cyclic groups."
7. If V is n-dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, then prove that T satisfies a polynomial of degree n over F.
8. If $u \in V_1$ is such that $\mathrm{uT}^{\mathrm{n_1-k}} = 0$ where $0 < k \leq n_1$, then prove that $u = u_0\, T^k$ for some $u_0 \in V_1$.
9. If M, of dimension m, is cyclic with respect to T, then prove that the dimension of MT^k is $m - k$ for all $k \leq m$.
10. Let V be an inner product space and T a self- adjoint linear operator on V. Then prove that each characteristic value of T is real, and characteristic vectors of T associated with distinct characteristic values are orthogonal.
11. Let f be a form on a finite-dimensional complex inner product space V. Then prove that there is an orthonormal basis for V in which the matrix of f is upper-triangular.
12. Let T be a linear operator on an n-dimensional vector space, prove that the characteristic and minimal polynomials for T have the same roots, except for multiplicities.

Section-C

Answer any THREE questions **(3x20=60)**

13. Let R be a Euclidean ring; then prove that any finitely generated R-module, M, is the direct sum of a finite number of cyclic submodules.
14. If $T\ E\ A(V)$ has all its characteristic roots in F, then prove that there exists a basis of V in which $m(T)$ is triangular.
15. Let $T\ E\ A(V)$ has all its distinct characteristic roots $\lambda_1, \lambda_2, \ldots \lambda_k$ in F then prove that there exist a basis of V for which the matrix of T is in Jordan form.
16. Let T be a linear operator on a finite dimensional vector space V. If f is the characteristic polynomial for T, then prove that $f(T) = 0$; in other words, the minimal polynomial divides the characteristic polynomial for T.
17. a) Let V be a finite-dimensional inner product space and f a form on V. Then prove that there is a unique linear operator T on V such that $f(\alpha, \beta) = (T\alpha|\beta)$ for all α, β in V and the map $f \rightarrow$ T is an isomorphism of the space of forms onto $L(V, V)$.

    b) Suppose the two matrices A, B in F_n are similar in k_n where K is an extension of F. Then prove that A and B are already similar in F_n.