STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2015–16 & thereafter)

SUBJECT CODE: 15MT/PC/FA44

M. Sc. DEGREE EXAMINATION, APRIL 2017
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : CORE
PAPER : FUNCTIONAL ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION - A (5x2=10)
ANSWER ALL THE QUESTIONS

1. State Jensen's inequality.
2. State the closed graph theorem.
3. Define the "Normed dual" of a space X.
4. Define "Weak convergence".
5. Define 'Normal' and 'Unitary' operators.

SECTION - B (5x6=30)
ANSWER ANY FIVE QUESTIONS

6. Let X and Y be normed spaces. If X is finite dimentional then prove that every linear map from X to Y is continuous.
7. Prove that a normed space X is a Banach space if and only if every absolutely summable series of elements in X is summable in X.
8. Let X be a separable normed space. Then prove that every bounded sequence in X' has a weak* convergent subsequence.
9. Let X be a reflexive normed space. If X' denotes the dual of X then prove X' is also reflexive.
10. State and prove the Schwarz inequality for inner product on a linear space X.
11. Let H be a Hilbert space, Let $A, B \in BL(H)$. If A and B are normal such that A commutes with B^* and B commutes with A^* then prove that $A + B$ and AB are normal.
12. Let H be a Hilbert space and let $A \in BL(H)$ Then prove that $\|A^*\| = \|A\|$ and $\|A^*A\| = \|A^2\| = \|AA^*\|$.

SECTION - C (3x20=60)
ANSWER ANY THREE QUESTIONS

13. Let X be a normed space. Prove the following conditions are equivalent.

(i) Every closed and bounded subset of X is compact.

(ii) The subset $\{x \in X: \|x\| \leq 1\}$ of X is compact

(iii) X is finite dimentional (10)

14. State and prove Hahn - Banach separation theorem.

15. State and prove the open mapping theorem.

16. a) State and prove the Bessal's inequality in inner product spaces. (10)

b) State and prove the Riesz representation theorem (10)

17. a) Let H be a Hilbert space. consider $A, B \in BL(H)$ and $k \in K$. Then prove the following:

(i) $(A+B)^* = A^* + B^*$, (ii) $(KA)^* = \overline{K}A^*$, (iii) $(A^*)^* = A$. further A is invertible if and only if A^* is invertible and $(A^*)^{-1} = (A^{-1})^*$. (10)

b) Prove that if $A \in BL(H)$ be self - adjoint. Then A or $-A$ is a positive operator if and only if $|< A(x), y >|^2 \leq < A(x), x >< A(y), y >$ for all $x, y \in H$. (10)