**STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086**
**(For candidates admitted from the academic year 2015–16 & thereafter)**

**SUBJECT CODE: 15MT/PC/DG44**

**M. Sc. DEGREE EXAMINATION, APRIL 2018**
**BRANCH I – MATHEMATICS**
**FOURTH SEMESTER**

**COURSE : CORE**
**PAPER : DIFFERENTIAL GEOMETRY**
**TIME : 3 HOURS** **MAX. MARKS : 100**

**SECTION - A (5x2=10)**
**ANSWER ALL THE QUESTIONS**

1. Find the speed for the logarithmic spiral.
2. When is a surface patch said to be regular.
3. Calculate the first fundamental form of the surface $\bar{\sigma}(u,v) = \bar{a} + u\bar{p} + v\bar{q}$.
4. Write the second fundamental form.
5. Define gaussian curvature.

**SECTION - B (5x6=30)**
**ANSWER ANY FIVE QUESTIONS**

.

6. Find the curvature of the circular helix $\gamma(\theta) = (acos\theta,\ asin\theta,\ b\theta), -\infty < \theta < \infty$ where $'a'$ and $'b'$ are constants.
7. Let $\gamma(t)$ be a regular curve is $\mathbb{R}^3$ with nowhere vanishing curvature, then show that its torsion is $= \frac{(\dot{\gamma}\times\ddot{\gamma}).\dddot{\gamma}}{\|\dot{\gamma}\times\ddot{\gamma}\|^2}$.
8. If $f: S_1 \to S_2$ be a diffeomorphism and if $\bar{\sigma}_1$ is an allowable surface patch on $S_1$ then prove that to $f \circ \bar{\sigma}_1$ is allowable surface patch on $S_2$.
9. Show that the area of a surface patch is unchanged by reparametrisation.
10. State and prove Meusnier's Theorem.
11. Show that a curve $\gamma$ on a surface is a geodesic if and only if for any part $\gamma(t) = \bar{\sigma}(u(t), v(t))$ of $\gamma$ contained in a surface patch $\bar{\sigma}$ of $S$ the following two equations are satisfied
$$\frac{d}{dt}(E\dot{u} + F\dot{v}) = \frac{1}{2}(E_u\dot{u}^2 + 2F_u\dot{u}\dot{v} + G_u\dot{v}^2)$$
$$\frac{d}{dt}(F\dot{u} + G\dot{v}) = \frac{1}{2}(E_u\dot{u}^2 + 2F_u\dot{u}\dot{v} + G_u\dot{v}^2)$$
where $Edu^2 + 2Fdudv + Gdv^2$ is the first fundamental form of $\bar{\sigma}$.
12. With the usual notation, show that
$$\bar{e}'_u.\ \bar{e}''_v - \bar{e}''_u\bar{e}'_v = \lambda'\mu'' - \lambda''\mu' = \alpha_v - \beta_u = \frac{LN-M^2}{(EG-F^2)^{1/2}}$$

**SECTION - C (3x20=60)**
**ANSWER ANY THREE QUESTIONS**

13. a) Prove that a parametric curve has a unit speed reparametrisation if and only if it is regular.
    b) State and prove the Frenet-Serret equation..

14. a) Show that the unit sphere $S^2$ is a smooth surface.
    b) Let $U$ and $\widetilde{U}$ be open subset of $\mathbb{R}^2$ and let $\bar{\sigma}: U \to \mathbb{R}^3$ be regular surface patches. If $\bar{\varphi}; \widetilde{U} \to U$ be an bijective smooth map with smooth inverse map $\varphi^{-1}: U \to \widetilde{U}$ then prove that $\tilde{\bar{\sigma}} = \bar{\sigma}.\bar{\varphi}$ is a regular surface patch.

15. a) Show that a diffeomorphism $f: S_1 \to S_2$ is a isometry if and only if for any surface patch $\bar{\sigma}_1$ of $S_1$, show that the patches $\bar{\sigma}_1$ and $f \circ \bar{\sigma}_1$ of $S_1$ and $S_2$ have the same first fundamental form.
    b) Show that any tangent developable is isometric to a plane.

16. a) If $\gamma(t) = \bar{\sigma}(u(t), v(t))$ is a unit-speed curve on a surface path $\bar{\sigma}$, prove that its normal curvature is given by $\kappa_n = L\dot{u}^2 + 2M\dot{u}\dot{v} + N\dot{v}^2$ where $Ldu^2 + 2Mdudv + Ndv^2$ is the second fundamental form of $\bar{\sigma}$.
    b) Let $\kappa_1$ and $\kappa_2$ be the principal curvatures at a point $P$ of a surface patch $\bar{\sigma}$ then show that
       i) $\kappa_1$ and $\kappa_2$ are real.
       ii) If $\kappa_1 = \kappa_2 = \kappa$ (say) then $\mathcal{F}_{II} = \kappa \mathcal{F}_I$ and hence every tangent vector to $\bar{\sigma}$ at $P$ is a principal vector .

17. a) Prove that a curve on a surface is a geodesic if and only if its geodesic curvature is zero everywhere.
    b) Show that any point of a surface of constant gaussian curvature is contained in a patch that is isometric to parts of a plane, a sphere or a pseudo sphere.