STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2015–16 & thereafter)

SUBJECT CODE: 15MT/PC/CI44

M. Sc. DEGREE EXAMINATION, APRIL 2018
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : CORE
PAPER : CALCULUS OF VARIATION AND INTEGRAL EQUATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION—A (5x2=10)

ANSWER ALL THE QUESTIONS

1. Show that for the functional $\int_0^{\pi/2}({y'}^2 - y^2)\,dx, y(0) = 0, y\left(\frac{\pi}{2}\right) = 1$ extremum can be achieved only on the curve y=sinx.
2. How diffracted rays are produced?
3. Verify that $y(x) = 1/2$ is a solution of the integral equation $\int_0^x \frac{y(t)}{\sqrt{x-t}} dt = \sqrt{x}$.
4. Define boundary value problem.
5. Show the homogeneous integral equation $y(x) - \lambda \int_0^1 (3x-2)ty(t)dt = 0$ has no eigen values.

SECTION—B (5x6=30)

ANSWER ANY FIVE QUESTIONS

6. State Brachistochrone problem and also solve it.
7. Find the extremum of the functional $I = \int_{x_1}^{x_2}\left({y'}^2 + {z'}^2 + 2yz\right)dx, y(0) = 0, z(0) = 0$ and the point (x_2, y_2, z_2) moves over the fixed plane $x = x_2$.
8. Find the extremals with corner points of the functional $I[y(x)] = \int_{x_1}^{x_2} {y'}^2(1-y')^2 dx$.
9. Define symmetric kernel and separable kernel. Also show that sin(x+t) is both symmetric and separable kernel.
10. Convert the differential equation $y'' + y = 0, y(0) = 0, y'(0) = 1$ into Volterra integral equation of the second kind.
11. Find the initial value problem corresponding to the integral equation $y(x) = 1 + \int_0^1 y(t)dt$.
12. Find the eigen values of the Fredholm equation $y(x) = \lambda \int_0^1 e^x e^t y(t)dt$.

SECTION—C (3x20=60)

ANSWER ANY THREE QUESTIONS

13. Obtain the necessary condition for the extremum of a functional of the form $\int_a^b F(x, y(x), y'(x), \ldots, y^{(n)}(x))dx$ where F is differentiable $n+2$ times with respect to all its arguments and hence determine the extremals of the functional $I[y(x)] = \int_{x_0}^{x_1}(2xy + (y'''(x))^2)dx$.

14. (a) Find the shortest distance between the parabola $y = x^2$ and the straight line $x - y = 5$.
(b) Find the shortest path from the point A(-2,3) to the point B(2,3) located in the region $y \leq x^2$.

15. (a) Show that the function $y(x) = \sin\left(\frac{\pi x}{2}\right)$ is a solution of the Fredholm integral equation $y(x) - \frac{\pi^2}{4}\int_0^1 K(x,t)y(t)\,dt = \frac{x}{2}$, where the kernel $K(x,t)$ is of the form

$$K(x,t) = \begin{cases} \frac{x(2-t)}{2}, 0 \leq x \leq t \\ \frac{t(2-x)}{2}, t \leq x \leq 1 \end{cases}$$

(b) Show that $y(x) = xe^x$ is a solution of the Volterra integral equation $y(x) = sinx + 2\int_0^x \cos(x-t)\, y(t)dt$.

16. Convert differential equation $y'' - sinxy' + e^x y = x$, $y(0) = 1, y'(0) = -1$ into a Volterra integral equation of the second kind and also convert the obtained Volterra integral equation in to the original differential equation.

17. Solve the following homogeneous Fredholm integral equation of the second kind $y(x) = \lambda\int_0^\pi \sin(x+t)y(t)dt$.