STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2015-16)

SUBJECT CODE : 15MT/MC/VL65

B. Sc. DEGREE EXAMINATION, APRIL 2018
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE : MAJOR CORE
PAPER : VECTOR SPACES AND LINEAR TRANSFORMATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL QUESTIONS. **(10X2=20)**

1. If V is a vector space over F then prove that $0v = 0$ for $v \in V$.
2. Define subspace of a vector space.
3. Prove that the vectors $(1, 2, 1), (2, 1, 0)$ and $(1, -1, 2)$ are linearly independent.
4. Prove that $F^{(n)}$ is isomorphic $F^{(n)}$ if and only if $n = m$.
5. If $dim_F V = m$ then prove that $dim_F Hom(V, V) = m^2$.
6. Define inner product space.
7. Prove that $\|\propto u\| = |\propto| \, \|u\|$.
8. Define invertible.
9. If $T \in A(V)$ and if $dim_F V = n$ then prove that T can have at most n distinct characteristic roots in F.
10. Find the eigen values of the matrix $A = \begin{bmatrix} 5 & -3 \\ 3 & -1 \end{bmatrix}$.

SECTION –B

ANSWER ANY FIVE QUESTIONS. **(5X8=40)**

11. If V is the internal direct sum of $U_1, U_s, \dots, U_n$ then prove that V is isomorphic to the external direct sum of $U_1, U_s, \dots, U_n$.
12. Prove that $L(S)$ is a subspace of V.
13. If V is finite dimensional over F then prove that any two bases of V have the same number of elements.
14. Let W be a subspace of V. Define $W^{\perp}$ and also prove that $W^{\perp}$ is a subspace of V.
15. State and prove Schwarz inequality.
16. Prove that the element $\lambda \in F$ is a characteristic root of $T \in A(V)$ if and only if for some $v \neq 0$ in V, $vT = \lambda v$.

17. Let $T:U \to V$ be a linear transformation. T is defined relative to bases $B=\{u_1,u_2,u_3\}$ and $B^{'}=\{v_1,v_2\}$ of U and V as follows.

$$T(u_1)=2v_1-v_2$$
$$T(u_2)=3v_1+2v_2$$
$$T(u_3)=v_1-4v_2$$

Find the matrix representation of T with respect to these bases and use this matrix to determine the image of the vector $u = 3u_1 + 2u_2 - u_3$.

SECTION –C

ANSWER ANY TWO QUESTIONS. **(2X20=40)**

18. (a) If F is a field of real numbers then show that the set of real – valued continuous functions on the closed interval $[0,1]$ forms a vector space over F.

(b) If V is finite dimensional and if W is a subspace of V then prove that W is finite dimensional, $\dim W \le \dim V$ and $\dim \frac{V}{W} = \dim V - \dim W.$

19. Prove that a finite-dimensional inner product space has an orthonormal set as a basis.

20. (a) If V is of finite dimensional over F then prove that $T \in A(V)$ is regular if and only if T maps V onto V.

(b) (i) Show that the following matrix A is diagonalizable.

(ii) Find a diagonal matrix D that is similar to A.

(iii) Determine the similarity transformation that diagonalizes A.

$$A=\begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix}$$

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