STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted from the academic year 2015-16 & thereafter)

SUBJECT CODE : 15MT/MC/SF45

B. Sc. DEGREE EXAMINATION, APRIL 2018
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : MAJOR CORE
PAPER : SEQUENCE, SERIES AND FOURIER SERIES
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL THE QUESTIONS: **(10×2=20)**

1. Define composition of functions.
2. Define cantor set.
3. Define a convergent sequence.
4. Give an example of an oscillating sequence.
5. Define a cauchy sequence.
6. Give an example of an alternating series.
7. Does the series $\sum_{n=1}^{\infty}\frac{2n}{n^2-4n+7}$ diverge?
8. State Ratio test.
9. Write the Fourier series.
10. Write the expression for a_n.

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5×8=40)**

11. If $f: A \to B$ and if $X \subset B,\ Y \subset B$, then prove that $f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$.

12. If A is any non empty subset of R that is bounded below, then show that A has a greatest lower bound in R.

13. Prove that if $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} s_n = M$ then $L = M$.

14. If $\{s_n\}_{n=1}^{\infty}$ is a sequence of real numbers which converges to L, then show that $\{s_n^2\}_{n=1}^{\infty}$ converges to L^2.

15. Prove that $\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)$ is divergent.

16. State and prove Abel's lemma.

17. Find a *sine* series for $f(x) = c$ in the range $0\ to\ \pi$.

SECTION – C

ANSWER ANY TWO QUESTIONS: (2×20=40)

18. a) If $A_1, A_2, \dots \dots,$ are countable sets, then prove that $\bigcup_{n=1}^{\infty} A_n$ is countable.

b) If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent, then show that $\{s_n\}_{n=1}^{\infty}$ is bounded.

19. a) Prove that any bounded sequence of real numbers has a convergent subsequence.

b) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

20. a) State and prove Dirichlet's test.

b) Express $f(x) = \frac{1}{2}(\pi - x)$ as a Fourier series with period 2π, to be valid in the interval to $0\ to\ 2\pi$.