STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2015-16 & thereafter)

SUBJECT CODE : 15MT/MC/ML24

B. Sc. DEGREE EXAMINATION, APRIL 2018
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : MAJOR CORE
PAPER : MULTIPLE INTEGRALS AND LAPLACE TRANSFORMS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION A

Answer All Questions: **10 x 2 = 20**

1. Evaluate : $\int_0^2 \int_0^y (x + y)\, dxdy$.
2. Integrate : $\int_0^1 \int_0^2 \int_0^3 xyz\ dxdydz$.
3. Evaluate : $\int_0^{\frac{\pi}{2}} \int_0^{acos\theta} r\, drd\theta$.
4. Given $x = rcos\theta\, , y = rsin\theta$, find $\frac{\partial (x, y)}{\partial (r, \theta)}$.
5. Using Beta and Gamma functions , evaluate $\int_0^1 x^4 \ (1 - x)^5\, dx$.
6. Define : Gamma function .
7. Derive the formula for $L\,(e^{-at})$.
8. Find : $L\,(t^3 - 2t + 5)$.
9. Evaluate : $L\,(t\, e^{-5t})$.
10. Find : $L^{-1}\left(\frac{1}{s^4}\right)$.

SECTION B

Answer Any Five Questions: **5 x 8 = 40**

11. Evaluate $\int\int xy\, dxdy$ taken over the positive quadrant of the circle $x^2 + y^2 = a^2$.
12. Given that $x + y = u\, , y = uv$, change the variables to $u\, , v$ in the integral.

    $\int\int [\, xy\,(1 - x - y)]^{\frac{1}{2}}\ dxdy$ taken over the area of the triangle with sides $x = 0\, , y = 0\, , x + y = 1$ and evaluate it.
13. Evaluate $\int_0^\infty e^{-x^2}\, dx$.
14. Find $L\,[f(t)]$ where $f(t) = 0 \ , 0 < t \le 2$

    $= 3 \ , t > 2$
15. Using Laplace Transforms, evaluate $\int_0^\infty \frac{e^{-t} - e^{-2t}}{t}\, dt$.
16. Change the order of integration in the integral $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dxdy$ and evaluate it.
17. Evaluate : $L^{-1}\left(\frac{s}{s^2 + 2s + 5}\right)$.

SECTION C

Answer Any Two Questions: **2 x 20= 40**

18. (a) Evaluate $\int\int xyz\, dxdydz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

(b) Evaluate $\int\int r\sqrt{a^2 - r^2}\; drd\theta$ over the upper half of the circle $r = acos\theta$.

(10 + 10)

19. (a) State and prove the relation between Beta and Gamma functions.

(b) Evaluate $\int_0^1 \frac{x^2\, dx}{\sqrt{1 - x^4}}$.

(10 + 10)

20. (a) Find $L\,(sin^2 2t)$.

(b) Evaluate : $L\,(t\; cos 3t\,)$

(c) Using Laplace Transforms solve :

$\frac{d^2y}{dt^2} - 10\frac{dy}{dt} + 24y = 24$ given $y(0) = 0$, $y^1(0) = 0$.

(5 + 5 + 10)