STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2015–16)

SUBJECT CODE : 15MT/MC/CA65

B. Sc. DEGREE EXAMINATION, APRIL 2018
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE : MAJOR CORE
PAPER : PRINCIPLES OF COMPLEX ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION-A

ANSWER ALL QUESTIONS: **10 X 2 = 20**

1. Show that the function $f(z)=\mathrm{Re}\,z$ is nowhere differentiable.
2. Show that the $u=\log\sqrt{x^2+y^2}$ is harmonic.
3. Is the mapping $w=\frac{1}{z}$ conformal?
4. Under the transformation $w=iz+i$ show that the half plane $x>0$ maps onto the half plane $v>1$.
5. Find the bilinear transformation which maps the points $z_1=0,\ z_2=-i$ and $z_3=-1$ into $w_1=i, w_2=1$and $w_3=0$ respectively.
6. Find the Taylor's series expansion of $f(z)=\frac{1}{z}$ about $z=1$.
7. Evaluate $\int_C \frac{dz}{z-3}$ where *C* is the circle $|z-2|=5$.
8. State Maximum modulus Theorem.
9. Show that for $f(z)=\frac{\sin z}{z}$, $z=0$ is a removable singularity.
10. Calculate the residue of $\frac{z+1}{z^2-2z}$ at its pole $z=2$.

SECTION-B

ANSWER ANY FIVE QUESTIONS: **5 X 8 = 40**

11. Let $f(z)=\begin{cases}\frac{xy}{x^2+y^2} & if\ \ z\neq 0\\ 0 & if\ \ z=0\end{cases}$ Show that the function satisfies C-R Equations at $z=0$, but not differentiable at that point.
12. Given $v(x,y)=x^4-6x^2y^2+y^4$. Find $f(z)=u(x,y)+iv(x,y)$ such that $f(z)$ is analytic.

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13. Find the image of the strip $2 < x < 3$ under $w = \frac{1}{z}$.

14. Define Flux and show that the temperature function satisfies Laplace equation at each interior point of the solid.

15. Expand $f(z) = \frac{z}{(z-1)(2-z)}$ in Laurent's series valid for

(a) $1 < |z| < 2$

(b) $|z-1| > 1$.

16. State and prove Liouville's theorem and hence prove fundamental theorem on algebra.

17. State Cauchy's Residue theorem and Evaluate $\int_C \frac{2+3\sin \pi z}{z(z-1)^2} dz$ where *C* is the square having the vertices $3+3i, 3-3i, -3+3i, -3-3i.$

SECTION-C

ANSWER ANY TWO QUESTIONS: 2 X20 = 40

18. (i) Derive the C-R equations of a function $f(z)$ in polar coordinates and find its derivative.

(ii) Find the image of the lines $x = c$ and $y = c, c \neq 0$ under the transformation $w = z^2$ and hence find the image of the *x*-axis and the *y*-axis. (10 +10)

19. (i) Determine the bilinear transformation which maps $0, 1, \infty$ to $i, -1, -i$ respectively. Show also that under this transformation the interior of the unit circle of the *z*-plane maps onto the left half of the *w*-plane.

(ii) State and prove the Taylor's theorem. (8 + 12)

20. (i) Evaluate: (a) $\int_C \frac{z dz}{z^2 - 1}$ where *C* is the positively oriented circle $|z| = 2$

(b) $\int_C \frac{z^3 dz}{(2z+i)^3}$ where *C* is the unit circle. (5 + 5)

(ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$ (10)