STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16 & thereafter)

SUBJECT CODE : 15MT/PC/TO34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2017
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : CORE
PAPER : TOPOLOGY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL THE QUESTIONS: **(5 x 2 = 10)**

1. Define subspace topology.
2. Define components of a space.
3. Define compact space.
4. When is a space said to be normal?
5. State Tychonoff theorem.

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5 x 6 = 30)**

6. Show that the topologies of $\mathbb{R}_\ell$ and $\mathbb{R}_k$ are strictly finer than the standard topology on $\mathbb{R}$, but are not comparable with one another.
7. If the sets C and D form a separation of X and if Y is a connected subspace of X, then prove that Y lies entirely with C or D.
8. Show that the image of a compact space under a continuous map in compact.
9. If X has a countable basis then prove the following
   a) every open covering of X contains a countable subcollection covering X.
   b) there exist a countable subset of X that is dense in X.
10. If $f : A \rightarrow \prod_{\alpha \in J} X_\alpha$ given by the equation $f(a) = (f_\alpha(a))_{\alpha \epsilon J}$ where $f_\alpha : A \rightarrow X_\alpha$ for each α. Let $\prod X_\alpha$ have the product topology. Then show that the function f is continuous if and only if for each function f_α is continuous.
11. Prove that $\bar{A} = A \cup A'$ where A is a subset of the topological space X and A' is the set of all limit points of A.
12. Show that a space X is locally connected if and only if for every open set U of X each component of U is open in X.

SECTION – C

ANSWER ANY THREE QUESTIONS: **(3 x 20 = 60)**

13. a) Show that the collection $D = \{B \times C / B \in \mathfrak{B},\ C \in \mathcal{C}\}$ is a basis for the topology on $X \times Y$ where $\mathfrak{B}$ is a basis for the topology on X and $\mathcal{C}$ is a basis for the topology on Y.

b) Prove that $x \in \bar{A}$ if and only if every basis element B containing x intersects A where A is a subset of a topological space X.

14. a) Show that intervals are connected in the order topology.

b) Show that the topologist *sine* curve is connected but not locally connected.

15. a) Prove that every closed subspace of a compact space is compact.

b) State and prove the Lebesque number lemma.

16. State and prove Urysohn metrization theorem.

17. a) If X and Y are topological spaces and if $f: X \rightarrow Y$ then show that the following statements are equivalent,

(i) f is continuous.

(ii) for every subset A of X, one has $f(\bar{A}) \subset \overline{f(A)}$.

(iii) for every closed set B in Y the set $f^{-1}(B)$ is closed in X.

b) State and prove the pasting lemma.

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