STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16 & thereafter)

SUBJECT CODE : 15MT/PC/RA14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2017
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : REAL ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 2 = 10)**
ANSWER ALL QUESTIONS

1. Let x and y be points in R^n. Then prove that $\|\boldsymbol{x}+\boldsymbol{y}\| \leq \|\boldsymbol{x}\| + \|\boldsymbol{y}\|$.
2. Define the convergence of a double sequence.
3. Define the Uniform Convergence of infinite series of functions.
4. When do you say that a function 'f' is differentiable at a point 'c' ?
5. If $f = u + iv$ is a complex valued function with a derivative at a point z in C, then prove that $J_f(z) = |f'(z)|^2$.

SECTION – B **(5 X 6 = 30)**
ANSWER ANY FIVE QUESTIONS

6. State and prove the Cantor Intersection theorem.
7. Prove that absolute convergence of $\prod(1 + a_n)$ implies convergence.
8. Give an example of a sequence of continuous functions with a discontinuous limit function.
9. State and prove the Cauchy condition for uniform convergence of a sequence of functions.
10. State and prove the Chain rule for differentiable functions.
11. State and prove the Mean-Value theorem for differentiable functions.
12. Let A be an open subset of R^n and assume that $f: A \to R^n$ is continuous and has finite partial derivatives $D_j f_i$ on A. If f is one-to-one on A and if $J_f(\boldsymbol{x}) \neq 0$ for each x in A, then prove that $f\ (A)$ is open.

SECTION – C **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

13. (a) State and prove the Representation theorem for open sets on the Real line.

    (b) Let S be a subset of R^n. Assume that every infinite subset of S has an accumulation point in S. Then prove that S is closed and bounded.

14. (a) State and prove the Merten's theorem on the Cauchy product of two series.

    (b) State and prove the Cauchy condition for convergence of an Infinite product.

15. (a) State and prove Weierstrass M-test.

    (b) State and prove the Bernstein's theorem on the convergence of the Taylor series.

16. If both partial derivatives $D_r f$ and $D_k f$ exist in an n-ball $B\ (c\ ;\ \delta\ )$ and if both are differentiable at 'c' , then prove that $D_{r,k}\ f(c) = D_{k,r}\ f(c)$.

17. State and prove the Implicit Function Theorem.

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