STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16& thereafter)

SUBJECT CODE : 15MT/PC/MA14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2017
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : MODERN ALGEBRA
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL THE QUESTIONS: (5 x 2 = 10)

1. Does there exist a subgroup of order 5 in a group of order 100? Justify your answer.
2. Define associates in a commutative ring and give examples in the ring of integers.
3. Show that the polynomial $x^3 - 3x^2 + 3$ is irreducible over the field of rational numbers.
4. If K is an extension field of a field F of degree p, where p is a prime, show that there is no intermediate fields between F and K.
5. Prove that the symmetric group S_3 is solvable.

SECTION – B

ANSWER ANY FIVE QUESTIONS: (5 x 6 = 30)

6. Prove that a group of order p^2 is abelian, where p is a prime.
7. Prove that any prime number of the form $4n + 1$ can be expressed asa sum of two squares.
8. State and prove the Eisenstein's criterion for irreducibility over the field of integers.
9. If a and b in K are algebraic over F of degrees m and n respectively and if m and n are relatively primes, prove that $F(a, b)$ is of degree mn over F.
10. If K is a finite extension of F, prove that $G(K, F)$ is a finite group and its order $o(G(K, F))$ satisfies $o(G(K, F)) \le [K : F]$.
11. Prove that any two Sylow p-subgroups of a finite group G are conjugates in G.
12. Prove that the roots of an irreducible polynomial over a field F of characteristic zero are all distinct.

SECTION – C

ANSWER ANY THREE QUESTIONS: (3 x 20 = 60)

13. Prove that every finite abelian group is the direct product of cyclic groups.
14. By proving the necessary results, prove the unique factorization theorem.
15. (a) State and prove division algorithm of polynomials over a field F.
    (b) If $f(x)$ and $g(x)$ are primitive polynomials, prove that $f(x)g(x)$ is also a primitive polynomial.
16. Prove that the number e is transcendental.
17. Prove that an extension field K of F is a normal extension of F if and only if K is splitting field of some polynomial over F.