STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16& thereafter)

SUBJECT CODE: 15MT/PC/GT34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2017
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : CORE
PAPER : GRAPH THEORY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 2 = 10)**

ANSWER ALL THE QUESTIONS

1. Define graph isomorphism.
2. Define perfect matching and give an example.
3. Prove that every critical graph is a block.
4. True or false: K_6 is a planar graph.
5. Define embedding of a graph.

SECTION – B **(5 X 6 = 30)**

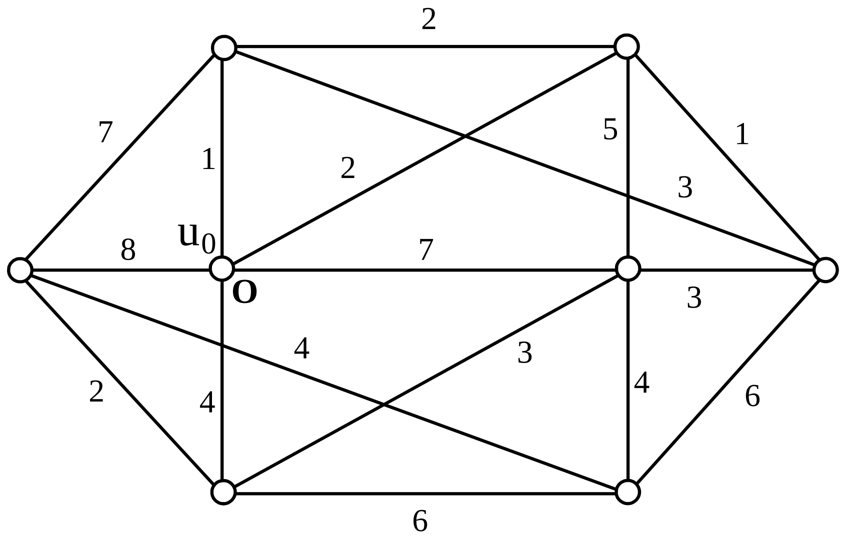
ANSWER ANY FIVE QUESTIONS

6. Prove that an edge is a cut-edge if and only if it belongs to no cycle.
7. State and prove Hall's theorem for bipartite graphs.
8. With usual notations prove that $\kappa \leq \kappa' \leq \delta$.
9. Prove that in a critical graph no vertex is a cut vertex. Hence prove that every critical graph is a block.
10. In any connected plane *(p,q)* graph $p \geq 3$ prove that $q \leq 3p-6$.Hence show that K_5 is not planar.
11. State and prove Euler's formula for planar graphs.
12. Write three equivalent definitions of de Bruijin digraph. Also draw B(2,3).

SECTION – C **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

13. Write Dijkstra's algorithm. Find shortest path from u_0 to all other vertices using Dijkstra's algorithm for the following graph.



14. a) Prove that a matching M in G is a maximum matching iff G contains no M – augmenting path.

    b) Prove that a graph is bipartite if and only if it contains no odd cycle.

(10 + 10)

15. a) State and prove Dirac theorem for vertex coloring.

    b)State and prove Brook's theorem. (10 + 10)

16. a) State and prove five colour theorem.

    b) State and prove Kuratowski'stheorem . (7 +13)

17. a) Write all the topological properties of Interconnected networks.

    b) Define Circulant network and list its characteristic features. Also draw$G(12, \{\pm 3\})$

(8+12)

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