STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16& thereafter)

SUBJECT CODE : 15MT/PC/DE14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2017
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : DIFFERENTIAL EQUATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 2 = 10)**

ANSWER ALL QUESTIONS

1. Define Linear dependence.
2. State Lipschitz condition.
3. Find a complete integral of $p^2 - y^2q = y^2 - x^2$.
4. Solve the B.V.P $\left(\frac{\partial u}{\partial x}\right) = 4\left(\frac{\partial u}{\partial y}\right)$if $u(0, y) = 8e^{-3y}$.
5. Give the general solution of three-dimensional Laplace equation by variable seperable method.

SECTION – B **(5 X 6 = 30)**

ANSWER ANY FIVE QUESTIONS

6. State and Prove Abel's formula.
7. State and prove Gronwall's inequality.
8. Compute the first three successive approximations of $x' = x^2, x(0) = 1$.
9. Find a complete integral of $yzp^2 - q = 0$.
10. Solve $x(y - x)r - (y^2 - x^2)\, s + y(y - x)t + (y + x)\,(p - x) = 2x + y + 2$ by canonical form.
11. If $P_m(t)$ and $P_n(t)$ are Legendre polynomials then prove that $\int_{-1}^{1} P_n(t)\, P_m(t)dt = 0$ if $m \neq n$.
12. Discuss the D'Alemberts solution of wave equation.

SECTION – C **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

13. Solve the Legendre equation $(1-t^2)x'' - 2t' + p(p+1)x = 0$.

14. State and prove Picard's theorem.

15. (a) Find a complete integral of $p^2x + q^2y = z$ by Charpit's method.

(b) Derive one dimensional wave equation.

16. (a) By using variable seperable method solve $\left(\frac{\partial u}{\partial t}\right) = C^2\left(\frac{\partial^2 u}{\partial x^2}\right)$, $t > 0, 0 \leq x \leq 1$

$$u(0,t) = 2$$

$$u(1,t) = 3$$

$$u(x,0) = x(1-x)$$

(b) The four edges of a thin squares plate of area π^2 are kept at temperature zero and the faces are perfectly insulated .The initial temperature is assumed to be $u(x,y,0) = xy(\pi - x)(\pi - y)$by applying the method of variable seperable to the two dimensional heat equation $u_t = C^2\nabla^2 u$ determine the temperature $u(x,y,t)$ in the plate.

17. Obtain steady temperature distribution in a semi circular plate of radius a ,insultated on both faces ,with its curved boundary kept at a constant temperature u_0 and its boundary diameter kept at zero temperature.

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