STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16 & thereafter)

SUBJECT CODE : 15MT/PC/CM14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2017
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : CONTINUUM MECHANICS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 2 = 10)**

ANSWER ALL THE QUESTIONS

1. State Cauchy's stress principle.
2. Write Lagrangian and Eulerian descriptions.
3. Define stream lines and path lines.
4. State the moment of momentum principle.
5. Define isotropy and anisotropy.

SECTION – B **(5 X 6 = 30)**

ANSWER ANY FIVE QUESTIONS

6. (i) Define deviator and spherical stress tensor.
   (ii) Show that for the stress tensor $\sigma_{ij} = \begin{pmatrix} 12 & 4 & 0 \\ 4 & 9 & -2 \\ 0 & -2 & 3 \end{pmatrix}$ the first invariant of the deviator is zero.
7. Derive Cauchy's deformation tensor.
8. Determine the velocity and acceleration of the particle of $p(1,3,2)$ when $t = 1$ for a velocity field $v = {x_1}^2 t\, \hat{e}_1 + x_2 t^2\, \hat{e}_2 + x_1 x_3 t\, \hat{e}_3$.
9. Obtain the Lagrangian differential form of the continuity equation.
10. Express the engineering constants υ and E in terms of Lame constants λ and μ.
11. Determine the Cauchy stress quadric at P for the following states.
    (i) uniform tension (ii) uniaxial tension
12. A flow is given by $v_1 = 0$, $v_2 = A(x_1 x_2 - {x_3}^2)e^{-Bt}$, $v_3 = A({x_2}^2 - x_1 x_3)e^{-Bt}$ where A and B are constants. Fine the rate of deformation tensor and the spin tensor for the point $P(1,0,3)$ when $t = 0$.

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SECTION – C (3 X 20 = 60)

ANSWER ANY THREE QUESTIONS

13. The stress tensor at a point P is given with respect to the axes $Ox_1x_2x_3$ by the values $\sigma_{ij} = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$. Determine the principal stress values and the principal stress directions represented by the axes $Ox_1^*x_2^*x_3^*$.

14. (i) Define the material deformation gradient F and the material displacement gradient .

(ii) Determine F and J for a given displacement field

$U = X_1{X_3}^2\,\hat{e}_1 + {X_1}^2X_2\,\hat{e}_2 + {X_2}^2X_3\,\hat{e}_3$. Verify the relation connecting F and J.

15. (i) What is material derivative? Write the material derivative of velocity and acceleration in Lagrangian and Eulerian form.

(ii) The motion of a continuum is given by $x_1 = A + \frac{e^{-B\lambda}}{\lambda}\sin\lambda(A+\omega t)$,

$x_2 = -B - \frac{e^{-B\lambda}}{\lambda}\cos\lambda(A+\omega t)$, $x_3 = X_3$. Find the particle paths and the velocity magnitude.

16. Derive the equilibrium equations.

17. Obtain the elastic coefficient matrix for an orthotrophic elastic continuum.