STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16& thereafter)

SUBJECT CODE: 15MT/PC/CA34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2017
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : CORE
PAPER : COMPLEX ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION–A

ANSWERALL QUESTIONS: **(5×2=10)**

1. Compute $\int_{|z|=1} |z-1|.|dz|$.
2. Prove that any harmonic function which depends only on r is of the form $a\, log r + b$ where a and b are constants.
3. Define infinite product of complex numbers.
4. Define equicontinuous family of functions.
5. Define free boundary arc.

SECTION–B

ANSWERANYFIVEQUESTIONS: **(5×6=30)**

6. State and prove Cauchy's theorem for a disk.
7. Prove that a non-constant harmonic function has neither a maximum nor a minimum in its region of definition.
8. State and prove a necessary condition for the absolute convergence of the product $\prod_1^\infty (1+a_n)$.
9. Show that every function which is meromorphic in the whole complex plane is the quotient of two entire functions.
10. Prove that a locally bounded family of analytic functions has a locally bounded derivative.
11. Prove that the functions $z = F(w)$ which map $|w| < 1$ conformally onto polygons with angles $\alpha_k \pi (k = 1,2,\dots,n)$ are of the form $F(w) = C \int_0^W \prod_{k=1}^n (w - w_k)^{-\beta_k} + C'$, where $\beta_k = 1 - \alpha_k$, w_k are points on the unit circle, C and C' are complex constants.
12. Discuss the boundary behaviour.

SECTION–C

ANSWERANYTHREEQUESTIONS: (3×20 =60)

13. (a) State and prove Cauchy's integral formula for higher derivatives.

(b) If the piecewise differentiable closed curve γ is does not pass through the point$a$ then show that $\int_{\gamma} \frac{dz}{z-a}$ is a multiple of $2\pi i$.

14. (a) State and prove generalized Cauchy's theorem.

(b) State and prove Poisson's formula.

15. (a) Show that $\zeta(s) = 2^s \pi^{s-1} sin \frac{\pi s}{2} \Gamma(1-s)\zeta(1-s)$.

(b) State and prove Jensen's formula.

16. (a) Prove that a family $\mathfrak{F}$ of analytic functions is normal with respect to $\mathbb{C}$ if and only if the functions in $\mathfrak{F}$ are uniformly bounded on every compact set.

(b) Show that a family of analytic or meromorphic functions f is normal in the classical sense if and only if the expressions$\rho(f) = \frac{2|f'(z)|}{1+|f(z)|^2}$. are locally bounded.

17. State and prove Riemann mapping theorem.