STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2004–05 & thereafter)

SUBJECT CODE : MT/MC/VL64

B. Sc. DEGREE EXAMINATION, APRIL 2009
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE : MAJOR CORE
PAPER : VECTOR SPACES AND LINEAR TRANSFORMATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL QUESTIONS : **(10 X 2 = 20)**

1. Define a subspace of a vector space.
2. If V is a vector space over F, then prove $(-\alpha)v = -\alpha v$, for $\alpha \in F$, $v \in V$.
3. Define a basis of a vector space.
4. Prove that $L(S)$ (Linear Span of S) is a subspace of V.
5. Give an example of an inner product space.
6. Prove that $\|\alpha v\| = |\alpha|\,\|v\|$.
7. Define a regular Linear transformation.
8. Define characteristic of a Linear transformation.
9. Define minimal polynomial of a transformation.
10. Prove that the vectors (1,0,0), (0,1,0) and (0,0,1) in $F^{(3)}$ are Linearly independent over F.

SECTION – B

ANSWER ANY FIVE QUESTIONS : **(5 X 8 = 40)**

11. If V is the internal direct sum of subspaces $U_1,...,U_n$ of V, then prove that V is isomorphic to the external direct sum of $U_1,...,U_n$.
12. If W_1 and W_2 are two subspaces of V, prove that $W_1 \cap W_2$ is a subspace of V.
13. State and prove Schwarz inequality.
14. Define $W^{\perp}$ of a subspace W of V. Prove that $W^{\perp}$ is a subspace of V. Also prove $W \cap W^{\perp} = \{0\}$.
15. Prove that Kernel of a homomorphism is a subspace of a vector space.
16. If V is a finite dimensional over F, then prove that $T \in A(V)$ is regular if and only if T maps V onto V.
17. If V is finite dimensional over F then prove for $S, T \in A(V)$.
    (i) $r(ST) \le r(T)$
    (ii) $r(ST) = r(TS) = r(T)$ for S regular in $A(V)$.

SECTION – C

ANSWER ANY TWO QUESTIONS : **(2 X 20= 40)**

18. a) If V is finite dimensional and if W is a subspace of V then prove that
    (i) W is finite dimensional and $\dim W \le \dim V$.
    (ii) $\dim \frac{V}{W} = \dim V - \dim W$.
    b) If A and B are finite dimensional subspaces of V , then prove that $(A+B)$ is finite dimensional and $\dim(A+B) = \dim A + \dim B - \dim(A \cap B)$
    (12+8)

19. State and prove Gram-Schmidt orthogonalization process. And give an example.
    (12+8)

20. a) If V is finite dimensional then prove $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not zero.
    b) If $T \in A(V)$ has all its characteristic roots in F , prove that there exists a basis of V in which the matrix of T is triangular. (10+10)