STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2004-05 & thereafter)

SUBJECT CODE : MT/MC/LS44

B. Sc. DEGREE EXAMINATION, APRIL 2009
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : MAJOR CORE
PAPER : LAPLACE TRANSFORMS, SEQUENCES AND SERIES
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL QUESTIONS: **(10 X 2 = 20)**

1. Prove that $L(t^n) = \frac{\Gamma(n+1)}{s^{n+1}}$.
2. Find $L^{-1}\left(\frac{1}{s(s+a)}\right)$.
3. Find $L^{-1}\left(\frac{s}{(s-b)^2+a^2}\right)$.
4. Find the Laplace transform of $f(t) = \begin{cases} e^{-t} & when\ 1 < t < 4 \\ 0 & when\ t > 4 \end{cases}$
5. Find a_o of the Fourier series for $f(x) = c$ in the range 0 to 2π.
6. State the least upper bound axiom.
7. Give an example of bounded set A of R whose glb and lub are both in $R - A$.
8. If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent then prove that $\{s_n\}_{n=1}^{\infty}$ is bounded.
9. If $\sum_{n=1}^{\infty} a_n$ is a convergent series then prove that $\lim_{n\to\infty} a_n = 0$.
10. State the ratio test for a series of real numbers.

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5 X 8 = 40)**

11. Find $L\left(\frac{\sin at}{t}\right)$
12. Find $L^{-1}\left(\frac{1}{(s+1)(s^2+1)}\right)$
13. Show that in the range $-\pi$ to π, e^x as a Fourier series is
$$e^x = \frac{\sinh \pi}{\pi}\{ \left\{1 + 2\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1}(\cos nx - \sin nx)\right\}$$

14. Prove that every Cauchy sequence is convergent.

15. Prove that if $\{s_n\}_{n=1}^{\infty}$ converges it cannot have more than one limit.

16. (i) If $0 < x < 1$ then prove that $\sum_{n=0}^{\infty} x^n$ converges to $\frac{1}{(1-x)}$.

(ii) If $x \geq 1$ then prove that $\sum_{n=0}^{\infty} x^n$ diverges.

17. Test the convergence of $\sum_{n=1}^{\infty} \frac{1}{n^p}$

SECTION – C

ANSWER ANY TWO QUESTIONS: (2 X 20= 40)

18. a) If $\{a_n\}_{n=1}^{\infty}$ is a sequence of positive numbers such that

(i) $a_1 \geq a_2 \geq .... \geq a_n \geq a_{n+1} \geq ...$ and

(ii) $\lim_{n\to\infty} a_n = 0$ then prove that the alternating series $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ is convergent.

b) State and prove nested-interval theorem.

19. a) Solve the differential equation $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 4y = t$ with the conditions $y(0) = 0,\ y'(0) = 1$.

b) Evaluate $\int_0^{\infty} \frac{e^{-t} - e^{-2t}}{t}\, dt$ (12+8)

20. A function $f(x)$ is defined within the range $(0,2\pi)$ by the relations

$$f(x) = \begin{cases} x & \textit{in the range } (0,\pi) \\ 2\pi - x & \textit{in the range } (\pi,2\pi) \end{cases}$$

Express $f(x)$ as a Fourier series in the range $(0,2\pi)$. Deduce that

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + ... = \frac{\pi^2}{8}$$