

STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2004–05 & thereafter)

SUBJECT CODE : MT/MC/CA64

B. Sc. DEGREE EXAMINATION, APRIL 2009
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE : MAJOR CORE
PAPER : COMPLEX ANALYSIS
TIME : 3 HOURS

MAX. MARKS : 100

SECTION – A

ANSWER ALL QUESTIONS :

(10 X 2 = 20)

1. Define: Analytic function.
2. State the C-R equations in polar co-ordinates.
3. Define: Harmonic function.
4. Find the critical points of the mapping $w = z + \frac{1}{z}$.
5. Define: Cross ratio.
6. Evaluate $\int_C \frac{dz}{z-4}$ where $C : |z| = 3$.
7. State Morera's theorem.
8. State Taylor's theorem.
9. Find the zeroes of $f(z) = (z-2i)^2(z+3)^3 e^z$.
10. State Cauchy's Residue Theorem.

SECTION – B

ANSWER ANY FIVE QUESTIONS :

(5 X 8 = 40)

11. Prove that $f(z) = e^x(\cos y + i \sin y)$ is differentiable at every point. Hence find $f'(z)$.
12. Find the bilinear transformation which maps. $z_1 = 2, z_2 = i, z_3 = -2$ onto $w_1 = 1, w_2 = i, w_3 = -1$ respectively.
13. Discuss the mapping $w = z^2$.
14. Evaluate $\int_C \frac{z}{z^2-1} dz$ where $C : |z| = 2$
15. State and prove the maximum modulus theorem.
16. Find the Laurent's series expansion for $f(z) = \frac{1}{(z-1)(z-2)}$ valid in the regions
(i) $|z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$
17. Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$.

SECTION – C

ANSWER ANY TWO QUESTIONS :

(2 X 20 = 40)

18. a) Derive the C-R equations in Cartesian form .
b) Given $u(x, y) = 2x - x^3 + 3xy^2$, find the analytic function having $u(x, y)$ as its real part. (12+8)
19. a) State and prove Cauchy's Integral Formula
b) Evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$ where $C : |z| = 2$. (12+8)
20. a) Using the method of contour integration evaluate $\int_0^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$.
b) Find the residue of $f(z) = \frac{e^z}{z^2(z^2+9)}$ at its poles. (12+8)

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