CHAPTER 1

INTRODUCTION

1.1 SQUEEZE FILM

A squeeze film is a fluid layer situated between surfaces which are approaching each other. The squeezing action of the approaching surfaces causes the fluid to flow towards less constrained surrounding. For very thin films, viscous forces offer a high resistance to such fluid motion, which in turn, tends to inhibit the approach of the bounding surfaces. In this way, a squeeze film serves to ward off potential contact between the surfaces. Squeeze films are encountered in numerous applications of lubrication technology.

The thesis presents the results of a study that has been made of one of the most elementary systems in continuum mechanics and the most elementary type of unsteady flow in lubrication theory; namely the problem of unsteady squeezing of a viscous incompressible fluid between two lubricated surfaces which approach each other with a normal velocity. Apart from the academic interest of the problem, the study of squeeze flows is important in some branches of both mechanical and chemical engineering, for instance in reciprocating engine bearing performance, in the unsteady loading of mechanical parts as in thrust bearings and in the study of adhesives. Lubrication engineers are concerned with the problem because of the similarity of the system to some types of load bearing. Others have found squeeze film between two flat surfaces ideal for use as a viscometer. An apparatus built with this purpose in mind, called a "parallel plate plastometer" is especially valuable for characterizing extremely viscous materials. Recently, the system has also become a subject of interest to rheologists who have been studying the behaviour of non-Newtonian fluids in such a plastometer (Leider and Bird 1974, Leider 1974, Brindley et al. 1976). We also cite some recent investigations of non-Newtonian flows in squeeze bearings and thrust bearings (Ramanaiah 1967a, 1979, Ramanaiah and Dubey 1975, 1977, Mahanti and Ramanaiah 1976, Ramanaiah and Priti Sarkar 1978, 1979).

Stefan (1874) in his study of squeeze films considered the case of parallel circular coaxial disks which moved only in the axial direction and made use of what later became known as the "lubrication approximations" in which the inertial terms of the Navier-Stokes equations are disregarded. Inspite of this, his result has been found to be quite useful. Needs (1940) demonstrated the experimental confirmation of the analysis at the low squeeze rates consistent with Stefan's assumption of no fluid inertia effects. Reynolds (1886) improved on Stefan's (1874) work by accommodating arbitrary surface geometry but confined his attention to the non-inertial case. The equations, Reynolds derived and integrated were applicable to the situation in which a Newtonian fluid of invariable properties was being slowly squeezed out of the space between two rigid, flat parallel plates with elliptical boundaries. The phenomena in fluid-film lubrication that have been successfully simulated by using an appropriate form of the Reynolds equation are numerous. In its most general formulation, the Reynolds equation accounts for arbitrary surface motion, side-leakage flow and variable viscosity and density. Indeed, even non-Newtonian effects can be described. The theory of elasto-hydrodynamic lubrication, which deals with the coupling of the Reynolds equation and the elasticity equation for the deformation of the bounding surfaces, is now a well established and powerful tool in describing the lubrication of non-conformal conjunctions. Thermal effects have also been incorporated into the analysis of both hydrodynamic and elastohydrodynamic lubrication regimes. These achievements have been possible mainly because the Reynolds equation constitutes a significant simplification of the governing equations for fluid flow.

Although the theory of squeeze films has been generally understood for sometime (Fuller 1956, Archibald 1956, Hays 1963, Beck et al. 1969), many of its important effects have been given thoughtful study only in recent times. Moore (1965) presents a review of the squeeze film literature through 1965. The interest in squeeze flow problems was in part sparked by the increase in operating speeds of the modern machines and the use of lubricants of low viscosity and relatively high density. These have motivated many recent studies that have revealed the importance of inertial effects and there has been a flurry of activity directed towards including the inertial terms in the analysis.

Two attempts to include the fluid inertial effects in analytical solutions of the squeeze film are those of Jackson (1962) and Pinkus and Sternlicht (1961). Neither of them included all the possible inertia terms in the momentum equations. However, there may be situations where the inertial effects may be significant and cannot be neglected on the assumption that they are small in comparison to viscous effects. Further, an order of magnitude analysis performed to find the order of magnitude of the ratios of the various terms in the Navier-Stokes equation revealed that all the inertia terms are of the same magnitude and if inertial effects are considered, then all the terms must be retained in the analysis.

Kuzma (1967) improved Jackson's (1962) analysis by including all inertial terms. He obtained expressions for the instantaneous pressure distribution and the load carrying capacity by using an iterative technique initiated by Jackson. He assumed that the squeeze Reynolds number is small and this formed the basis of the iteration scheme. His results are in good agreement with the experimental results.

Ishizawa (1966) made a comprehensive and elegant study of the flow caused by fairly general motion of the plates and presented a theoretical investigation of unsteady laminar flow of a viscous incompressible fluid between two parallel walls in motion normal to their own surfaces independent of each other and arbitrary with respect to time. The exact solution of the Navier-Stokes equations has been obtained as a multifold series in terms of an infinite set of time dependent non-dimensional parameters for the case of axisymmetric radial flow. As an application, a detailed numerical study has been made of the fundamental case when the walls perform reciprocating harmonic oscillations with finite amplitude.

Evelyn Chandrasekharan and, Ramanaiah (1983) have investigated the unsteady squeeze flow of a viscous incompressible fluid between rectangular plates and between circular plates by taking into account the local and convective inertial effects. The sinkage relation between the thickness of the film and time has been obtained for various values of the Reynolds number, for a given load on the upper plate. The departure from the classical inertialess solution is exhibited for various values of the two dimensionless parameters involved, one characterizing the load and the other gravity.

1.2 OSCILLATING SQUEEZE FLOWS

An interesting and useful class of squeeze flow occurs when one of the surfaces undergoes sinusoidal oscillation. Such a flow has been treated by various authors (Kuhn and Yates 1964, Hunt 1966, Terrill 1969, Modest and Tichy 1978) and can be observed when hydrodynamic slider bearings are subject to periodic vibrations in the load or in the periodic motion of gear teeth. For these problems, the variation of the pressure is of particular interest since it could produce wear on the surfaces; further, if the change in pressure is such that cavitation occurs then extensive damage could be produced.

Kuhn and Yates (1964) obtained a solution for the pressure of the fluid between the plates by including inertia terms and they also performed some experimental work which agreed with their solution. However, their solution did not satisfy the boundary conditions on the oscillating disk, the position of which is time dependent as they failed to take account of this time-dependent boundary correctly. Hunt (1966) obtained the maximum and minimum pressures in the fluid, neglecting inertia terms and showed that they were dependent on the amplitude of the oscillation and it agreed well with his experimental work.

Tichy and Modest (1978) obtained an analytical solution for oscillating squeeze films with arbitrary two dimensional geometry. This result is significant in that the contribution to load or pressure due to squeezing motion can be evaluated for a variety of practical squeeze film geometries. This load helps in maintaining the separation of the surfaces during the approach phase of the motion.

Terrill (1969) obtained an analytic solution of the Navier-Stokes equations for the case of parallel circular disks in which the disk oscillation is normal to its plane, the oscillation amplitude is much smaller than the film thickness, and the film thickness is in turn much smaller than the disk radius. He showed that the solution depends on two parameters; the non-dimensional amplitude of the oscillation of the disk and a Reynolds number related to the maximum velocity of the vibrating disk.

Tichy and Winer (1970) investigated inertial considerations in parallel circular squeeze film bearings and obtained a second order regular perturbation solution for squeeze film flow by including all the inertial terms in the momentum equation. Experimental measurements of pressure in a squeeze film at high squeeze rates are shown to be in good agreement with the solution. The solution is particularly useful in assessing the extent to which fluid inertial effects would cause the classical lubrication theory to be in error for any particular squeeze film.

1.3 SIMILARITY SOLUTIONS IN HYDRODYNAMIC SQUEEZE FLOWS

Wang (1976) showed that the unsteady Navier-Stokes equations governing the squeezing of a fluid between two plates admit a similarity solution if the normal velocity is proportional to $(1-\alpha t)^{-1/2}$ where α is a constant of dimension $[\mathrm{Time}^{-1}]$ and this is confirmed by group theoretic approach by Phan-Thien (1981) for the two dimensional case and Usha and Uma (1991) for the axisymmetric case. Wang's solution is governed by a non-dimensional parameter that characterises unsteadiness. Asymptotic solutions for small and large values of the parameter have been obtained and are found to agree well with those obtained by numerical integration.

Wang and Watson (1979) also considered the squeezing of a viscous fluid between elliptic plates and obtained exact similarity solutions when the gap width varies as the square root of time. The resulting non-linear boundary value problem is solved by perturbation theory and also integrated numerically by a new homotopy method. He showed that non-unique solutions exist for the separation of plates.

A similar solution for the unsteady flows produced by a single contraction or expansion of the wall of a semi-infinite circular pipe has been investigated by Uchida and Aoki (1977) with special attention to the non-linear characteristics of such unsteady flows. This flow model is motivated by physiological pumping like that in the left ventricle, in valved veins and in their bronchial tubes.

1.4 SQUEEZE FLOW FOR DIFFERENT STATES

Braking is an important mechanical process whereby excess kinetic energy is dissipated into heat. The most common braking method is through friction between solids. Although quite efficient, this method causes heavy irreversible wear and tear on the solid parts in contact. Wang (1981) has studied theoretically a novel way to achieve braking namely by introducing a very viscous fluid between the moving solid parts. The wear and tear are minimal since no solid to solid contact is required. Also, most of the energy is dissipated by the viscous fluid which can be easily replenished. Another instance of such hydrodynamic braking occurs when a fluid is inadvertently introduced between brakes designed for dry friction, such as the skidding of automobile tyres on a wet pavement. Wang (1981) considered the two dimensional case with the bottom plate moving laterally in the horizontal direction with a time dependent velocity and the top plate having vertical downward motion, squeezing the viscous fluid between the plates which in turn increases the shear drag on the bottom plate. The solution is obtained as a power series in terms of a squeeze number that represents the relative importance between unsteady forces and the viscous forces, for small values of the squeeze number. Braking characteristics are obtained for different states; constant velocity squeezing, constant force squeezing and constant power squeezing.

The squeezing of a viscous fluid from a circular tube with a shrinking diameter occurs not only in mechanical processes such as the release of air from long balloon but more important, in the biological shifting of body fluids by vaso constriction. For instance, if mammals are subjected to psychological and physiological stress, the blood vessels of the peripheral and splanchnic vascular beds constrict and squeeze blood into the high priority organs like brain and heart. Wang (1980) investigated arbitrary squeezing of a fluid from a tube at low squeeze numbers and found that the radius of the tube decays exponentially to zero for the constant power squeezing state.

Jones and Wilson (1975) utilized a method of matched asymptotic expansion in order to obtain a uniformly valid asymptotic solution which includes the contribution of fluid inertia for the special case of a fixed lower plate and an upper plate which moves impulsively from rest to a constant velocity.

Hamza and MacDonald (1981) analysed the case when one surface is fixed and the other is rapidly accelerated from a state of rest to a state of uniform motion. Their analysis is based in part on linear theory and in substance on a finite difference analysis of the full nonlinear equations of motion.

The problem of an object moving under the action of a prescribed body force of constant magnitude towards a fixed lower plane boundary (constant force squeezing state) is more closely related to most application and has been investigated by Weinbaum et al. (1985), Lawrence et al. (1985) and Lawrence and Weinbaum (1988). Recently, Yang and Leal (1993) have examined the time-dependent draining of fluid layer between two rigid parallel surfaces for the thin-film lubricant limit by considering the case in which an initially stationary object with a circular plane lower surface begins to suddenly move under the action of a constant applied force towards a parallel plane wall when the inertia of the object and that of the intervening fluid in the gap are not negligible.

Inspite of the fact that perturbation solution to the circular squeeze film problem by definition do not apply in the limit of $t \rightarrow 0$ (Jones and Wilson 1975), the first order perturbation solutions to the circular squeeze-film force have been shown by Kuzma (1967) to agree with experiment and by Grimm (1976) to agree with complete numerical solution. The success of the first order perturbation approximations in predicting squeeze film forces for arbitrary motion of circular plates and the ability of such a solution to be linked easily to general lumped parameter analysis have given confidence in the applicability of the solutions presented for the circular squeeze film. Also, the second order perturbation solution was included in a wide-ranging study of the squeeze film problem by Ishizawa (1966). Ishizawa (1966) drew attention to the rapid decrease in the magnitude of the universal functions of the velocity distribution that occur in the perturbation expansion which indicate the convergence of the power series expansion used in the analysis.

1.5 SQUEEZE FLOW WITH CENTRIFUGAL EFFECTS

Centrifugal effects in lubrication have received considerable attention (Allen and McKillop 1970, Pinkus and Lund 1981, Missimer and Johnson 1982) due to the important roles they play in many industrial applications. Some examples include thrust bearings, hydrodynamic face seals, oil lubricated brakes, wet clutches in automatic transmissions and some journal bearings. Hamza and MacDonald (1984) obtained a similarity solution to the motion of a fluid film squeezed between two parallel disks spaced a distance proportional to the square root of time and rotating with angular velocities proportional to the reciprocal of time. They have been able to investigate the effects in combined sliding and squeezing flow. They noted that, for a fixed squeeze Reynolds number, the load decreases with increase of rotation Reynolds number and that the torques are more sensitive to changes in the squeeze Reynolds number than to changes in the rotation Reynolds number.

Hamza (1985) has also examined the case when the lubricant is squeezed between two rotating parallel plane surfaces which are impulsively set in motion and studied the way inwhich the lubricant flow pattern, the load capacity and the friction torque change with time, the squeeze Reynolds number and the rotation Reynolds number. Hamza's results indicate that the effect of increasing the rate of squeezing with a fixed rate of rotation is to increase the torque exerted on the fluid by the rotating surface and the effect of rotation with a fixed rate of squeezing is to decrease the normal forces exerted on the surfaces.

1.6 THE MAGNETOHYDRODYNAMIC SQUEEZE FILM

In recent years, the magnetohydrodynamic lubrication phenomenon has received considerable attention and many studies of the problem have been motivated by the increased use of liquid metal lubricants in high temperature bearings. Authors who have investigated this phenomenon include Hughes and Elco (1962), Kuzma et al. (1964), Krieger et al. (1967), Kamiyama (1969), Ramanaiah (1967b, 1968) and Hamza (1988, 1989). The first five authors considered the magnetic force term but neglected some or all the inertia terms in the governing equations of motion. Hamza (1988) analysed the motion of an electrically conducting fluid film squeezed between two parallel disks in the presence of a magnetic field applied perpendicular to the disk. Analytical solutions through the use of a regular perturbation scheme are obtained for the constant velocity squeezing state. The results show that the electromagnetic forces increased the load carrying capacity considerably.

Hamza (1989) obtained a similarity solution for a flow between two parallel disks when the disks are separated by a distance proportional to the square root of time and a magnetic field proportional to the inverse square root of time is applied perpendicular to the disks. Approximate analytic solutions are given and a numerical solution to the resulting nonlinear ordinary differential equations is presented. The effects of the magnetic forces on the velocity profiles and on the normal forces which the fluid exerts on the disks are studied. It has been shown that by increasing the magnetic force a considerable increase in the load can be achieved.

Usha and Sangita Vasudevan (1993) investigated the centrifugal inertia effects in the magnetohydrodynamic squeeze film and obtained a similarity solution.

It has been observed that the torque exerted on the fluid by the rotating surface increases with the increase of rate of squeezing for a fixed rate of rotation and the normal forces exerted on the surfaces decrease with the increase of rotation for a fixed rate of squeezing.

1.7 THERMAL BEHAVIOUR OF SQUEEZE FILMS

Heat transfer effects in lubrication have long been recognized as important in determining bearing performance characteristics. Heat generation by viscous dissipation and heat transfer from or to bearing surfaces both serve to create large temperature variations in the lubricant film. The most important consequence of these effects is a potential decrease in load carrying capacity when the viscosity drops. In addition, for those cases in which the lubricant serves not only as a load carrying medium, but also to cool the machine parts, knowledge of the temperature field is necessary to predict lubricant requirements.

Early attempts at understanding thermal effects neglected the temperature variation across the lubricating film and generally, assumptions were made about the relative magnitudes of convection along the film, conduction to bearing surfaces and viscous dissipation. A complete solution, including variation of temperature across the film and viscosity dependence on temperature was first obtained by Zienkiewicz (1957). A thorough review of the extensive literature concerned with thermal aspects of hydrodynamic lubrication has been presented by Pinkus and Wilcock (1979).

An early work concerned with the thermal behaviour of squeeze films is that of Gould (1967). The parallel disk problem is considered for a general squeezing motion and a numerical solution is presented along with experimental results. Tichy and Smith (1983) have obtained an analytic solution for the temperature field in an oscillating squeeze film flow. The usual lubrication assumptions of negligible fluid inertia, thin film, low curvature for two dimensional flow are invoked, and the oscillations are assumed to be small relative to the gap size. In addition, a constant viscosity condition is assumed, which, according to order-of-magnitude analysis is applicable to many practical cases.

In the unsteady laminar flow between two parallel disks with arbitrarily varying gapwidth, heat is generated in the gap by the viscous friction of the fluid flow, ceaselessly throughout the wall motion. Usually, the generated heat will be removed mainly by cooling through the wall surface. In fact, when the change in the gap width is purely periodic, there is no convective flow which carries the generated heat away from the gap. As a result, in practical problems the temperature of the fluid rises to a high degree. Ishizawa (1966) analysed the change in the temperature distribution when the walls perform harmonic oscillations with finite amplitude by solving an exact thermal energy equation assuming that the physical constants of the fluid are independent of temperature and pressure. The solution of the thermal energy equation has been obtained as a multifold series interms of an infinite set of non-dimensional parameters that govern the motion of the plates.

1.8 ANNULAR SQUEEZE FILMS

Analysis of laminar flow in a squeeze film between plane annular disks has been obtained by Archibald (1956). Archibald's analysis does not include the effect of fluid inertia. Theoretical and experimental study of Newtonian squeeze film between fixed and rotating annuli was carried out by Allen an McKillop (1970). They considered only the centrifugal inertia and neglected both local and convective fluid inertia. Turns (1983) analysed the inertial effects in annular squeeze film by a successive approximation technique. He used the velocity distribution obtained from the squeeze flow problem without inertial effects as a basic solution and obtained the first order solution. An expression for the force generated by the fluid is developed and is coupled to the equation of motion for the annular plate and the results are presented from the numerical integration of the governing equations for the plate motion. Elkouh (1984) investigated the squeeze flow problem between two plane non-rotating annuli by including both the local and convective inertia effects. A solution of the equations of motion has been obtained in terms of an infinite set of dimensionless time dependent parameters and is valid for small values of these parameters. First order expressions for the pressure and load capacity are presented and compared with the results based on the assumption of inertialess flow.

1.9 SCOPE OF THE PRESENT WORK

The main objective is to examine the squeezing of a viscous incompressible fluid between two surfaces, the gapwidth of which varies arbitrarily with time. The governing equations are Navier-Stokes equations with the equation of continuity which are transformed either to a nonlinear partial differential equation (chapters 2, 3 and 5) by the use of transformations relevant to the problem under investigation and approximate analytic solutions are obtained by using regular perturbation theory as a power series in terms of a single non-dimensional squeeze number or to a nonlinear ordinary differential equation (chapter 4) that is numerically integrated using a shooting technique. The boundary conditions on the velocity are the no-slip condition; and the pressure is obtained by integrating the equations of motion and using a condition on the pressure at the edge of the upper surface. In chapter 6, a multifold series solution of the resulting partial differential equation is obtained in terms of an infinite set of non-dimensional time-dependent parameters for squeeze flow between elliptic plates.

The changes in the lubricant flow pattern, load carrying capacity, friction torque (Chapter 3) with the squeeze Reynolds number, rotation Reynolds number (Chapter 3) and time are investigated for circular, annular and elliptic squeeze films for different states: harmonically oscillating plates, plates with velocity varying inversely with square root of time, constant velocity squeeze, constant force squeeze and constant power squeeze. The effect of mass transfer on the magnetohydrodynamic squeeze film is examined in chapter 4.

The last chapter is devoted to the study of axisymmetric motion of a liquid film on an unsteady stretching surface.