STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted from the academic year 2015-16)

SUBJECT CODE : 15MT/MC/SF45

B. Sc. DEGREE EXAMINATION, APRIL 2017
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : MAJOR CORE
PAPER : SEQUENCE, SERIES AND FOURIER SERIES
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL THE QUESTIONS: **(10×2=20)**

1. If A is the set of letters in the word "trivial" and B is the set of letters in the word "difficult". Find $A-B$ and $A\cap B.$
2. Find the inverse of the function $f:R\rightarrow R$ defined by $f(x)=4x+5, x\in R.$
3. Define limit of a sequence.
4. Write any two subsequences of a sequence of integers.
5. Define Cauchy sequence.
6. Give an example of a conditionally convergent series.
7. Examine the convergence of $\sum_{n=1}^{\infty}\frac{1}{2n+5}$
8. Write Abel's lemma.
9. Define even function and give an example.
10. Find the Fourier coefficient a_0 for $f(x)=\sin x$ in the interval $(0,\pi)$.

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5×8=40)**

11. If $f:A\rightarrow B$ and if $X\subset B, Y\subset B,$ Prove that $f^{-1}(X\cup Y)=f^{-1}(X)\cup f^{-1}(Y)$.
12. Prove that countable union of countable sets is countable.
13. If the sequence $s_n\ {}_{n=0}^{\infty}$ of real numbers is convergent to L, Prove that $s_n\ {}_{n=0}^{\infty}$ can not converge to a limit distinct from L.
14. Prove that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$ is convergent.
15. Examine the convergence of $\sum_{n=0}^{\infty}\frac{n^3+1}{2^n+1}$.

16. If $a_n{}_{n=1}^{\infty}$ is a sequence of real numbers whose partial sums $s_n = \sum_{k=1}^{n} a_k$ form a bounded sequence and if $b_n{}_{n=1}^{\infty}$ is a non increasing sequence of non negative numbers which converges to 0, Prove that $\sum_{k=1}^{\infty} a_k b_k$ converges.

17. Find the Fourier sine series of $f(x) = e^x$ in $(0, \pi)$.

SECTION – C

ANSWER ANY TWO QUESTIONS: **(2×20=40)**

18. a) Prove that the set of rational numbers in $[0,1]$ is uncountable and deduce that the set of all real numbers is uncountable.

b) Prove that a non decreasing sequence which is bounded above is convergent.

(12+8)

19. a) Prove that a sequence of real numbers is convergent if and only if it is a Cauchy sequence.

b) State and establish the ratio test. (6+14)

20. Determine the Fourier series expansion for $f(x) = x + x^2$ in $(-\pi, \pi)$ and hence deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.