# STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600086 

(For candidates admitted from the academic year 2015-16)
SUBJECT CODE : 15MT/AC/ST45

## B. Sc. DEGREE EXAMINATION, APRIL 2017 <br> BRANCH I - MATHEMATICS FOURTH SEMESTER

| COURSE | $:$ ALLIED CORE |
| :--- | :--- |
| PAPER | $:$ MATHEMATICAL STATISTICS - II |
| TIME | $: 3$ HOURS |

SECTION - A

## ANSWER ALL THE QUESTIONS:

$(10 \times 2=20)$

1. Write down the steps involved in the process of sampling.
2. Define standard error.
3. State any two properties of $\chi^{2}$ distribution.
4. Define $t$-distribution.
5. When do you say that an estimator is a good estimator?
6. Estimate the parameter $p$ of the binomial distribution by the method of moments.
7. Define confidence interval.
8. A research worker wants to determine the average time it takes a mechanic to rotate the tyres of a car, and she wants to be able to assert with 95 percent confidence that the mean of her sample is off by at most 0.05 minutes. If she can presume from past experience that $\sigma=1.6$ minutes, how large a sample will she have to take?
9. Define parametric test.
10. State any two applications of $F$-distribution.

> SECTION - B

## ANSWER ANY FIVE QUESTIONS:

11. A machine puts out 16 imperfect articles in a sample of 500 . After machine is overhauled, it puts out 3 imperfect articles in a batch of 100 . Has the machine improved?
12. Prove that all the moments of odd order about the origin vanish in Student $t$-distribution.
13. Define likelihood function and state the properties of maximum likelihood estimator.
14. In a random sample of size 500 from a population having variance 16 , the mean is found to be 20 . In another sample of size 400 having variance 25 , the mean is 15 . Construct a 95 percent confidence interval for the difference of sample means.
15. Following are the heights of ten individuals chosen randomly from a normal population: $63,63,66,67,68,69,70,71$ and 71 inches. From the given data, discuss the suggestion that the mean height in the population is 66 inches.
16. Define Fisher's $z$-distribution and derive its characteristic function.
17. If $y_{1}, y_{2}$ and $y_{3}$ constitute a random sample of size three from normal population with the mean $\mu$ and the variance $\sigma^{2}$, find the efficiency of $\left(y_{1}+2 y_{2}+y_{3}\right) / 4$ relative to $\left(y_{1}+y_{2}+y_{3}\right) / 3$.

## SECTION - C

## ANSWER ANY TWO QUESTIONS:

$(2 \times 20=40)$
18. (a) Show that the mean and standard error of sample mean $\bar{y}$ from simple samples of size $n$ are $E(\bar{y})=\mu$ and $S . E .(\bar{y})=\sigma / n$ where $\mu$ and $\sigma$ denote the mean and standard deviation of the population.
(b) If $\chi_{1}^{2}$ and $\chi_{2}^{2}$ are independent $\chi^{2}$ variates with $n_{1}$ and $n_{2}$ degrees of freedom respectively, prove that $\chi^{2}=\chi_{1}^{2}+\chi_{2}^{2}$ is a $\chi^{2}$ variate with $n_{1}+n_{2}$ degrees of freedom and $T^{2}=\chi_{1}^{2} / \chi_{2}^{2}$ is a $\beta_{2}\left(\frac{n_{1}}{2}, \frac{n_{2}}{2}\right)$ variate.
19. (a) State and prove Rao-Blackwell's theorem.
(b) Let $X_{1}, X_{2}, \ldots, X_{n}$ be a sample from a normal population with unknown mean $\mu$ and unknown variance $\sigma^{2}$. Find the confidence interval and in particular find the 95 percent confidence interval for the data $5,8.5,12,15,7,9,7.5,6.5$ and 10.5 .
20. (a) Following is the contingency table for production in three shifts in a factory and number of defective goods turnout. Is it possible that the number of defective goods depends on the shift run by the factory?

|  | No. of defective goods in three weeks |  |  | Total |
| :---: | :---: | :---: | :---: | :---: |
| Shift | $1^{\text {st }}$ week | $2^{\text {nd }}$ week | $3^{\text {rd }}$ week |  |
| I | 15 | 5 | 20 | 40 |
| II | 20 | 10 | 20 | 50 |
| III | 25 | 15 | 20 | 60 |
| Total | 60 | 30 | 60 | 150 |

(b) The nicotine content in milligrams of two samples of tobacco was found to be as follows.

| Sample A | 24 | 27 | 26 | 21 | 25 | --- |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Sample B | 27 | 30 | 28 | 31 | 22 | 36 |

Can it be said that two sample come from normal populations having the same mean?

