

STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086  
(For candidates admitted from the academic year 2011–12& thereafter)

SUBJECT CODE: 11MT/ME/SM63

B. Sc. DEGREE EXAMINATION, APRIL 2017  
BRANCH I – MATHEMATICS  
SIXTH SEMESTER

COURSE : MAJOR ELECTIVE  
PAPER : SPECIAL TOPICS IN MECHANICS  
TIME : 3 HOURS

MAX. MARKS: 100

SECTION-A

ANSWER ALL QUESTIONS:

10 X 2 = 20

1. Define :centre of mass.
2. Write the centre of mass for an arc of a circle of radius  $a$  subtending an angle  $2\alpha$  at the centre.
3. Prove that  $T^2 - T_0^2 = W^2$  for the common catenary, where  $T$  is the tension at any point of the string,  $T_0$  is the tension at the lowest point, and  $W$  is the weight of a length  $s$  of the string.
4. Write the intrinsic equation of the common catenary.
5. State Newton's Experimental Law.
6. Define : impulsive force.
7. Write the moment of inertia of a circular lamina of radius  $a$  about a diameter.
8. Define : Moment of inertia.
9. Write the expression for the angular momentum of a rigid body rotating about a fixed axis.
10. What is the length of a simple equivalent pendulum?

SECTION-B

ANSWER ANY FIVE QUESTIONS:

5 X 8 = 40

11. Derive the centre of gravity of a solid cone of uniform density.
12. A cylinder and a cone have their equal common base joined together. Find the ratio of the height of the cone to the height of the cylinder so that the centre of gravity of the combination may coincide with the centre of the common base.
13. A telegraph wire stretched between two poles distance  $a$  apart, sags  $n$  feet in the middle. Prove that terminal tension is approximately  $w\left(\frac{a^2}{8n} + \frac{7n}{6}\right)$ , where  $w$  is the weight per unit length of the wire.
14. An endless uniform chain rests in equilibrium over a smooth pulley and is in contact with it over three quarters of the circumference. Show that the length of the free portion is  $\frac{\sqrt{2}}{\log(\sqrt{2}+1)}$  times the radius of the pulley.

15. Find the moment of inertia of a circular plate about a tangent line.
16. A ball impinges directly on a second ball of twice its mass which is moving in the same direction as the first but with one-seventh of its velocity. Given that  $e = \frac{3}{4}$ , show that the first ball will come to rest after the impact.
17. Find the minimum time of oscillation of a compound pendulum and prove that in that case, the length of the simple equivalent pendulum is twice the radius of gyration (about a line through the centre of gravity  $G$  and parallel to the axis of rotation).

### SECTION-C

ANSWER ANY TWO QUESTIONS:

2 X 20 = 40

18. (a) Derive the centre of gravity of a solid hemisphere of radius  $a$ .
- (b) A particle falls from a height  $h$  upon a smooth fixed horizontal plane. If  $e$  be the coefficient of restitution, show that the time  $T$  that elapses before the particle has ceased to rebound is  $\left\{ \frac{2h}{g} \right\}^{1/2} \frac{1+e}{1-e}$ . (10 + 10)
19. (a) Derive the cartesian equation of the common catenary.
- (b) A uniform chain of length  $2l$  hangs between two points  $A$  and  $B$  in the same horizontal line. The tension both at  $A$  and at  $B$  is five times the tension at the lowest point. Prove that the horizontal distance between  $A$  and  $B$  is  $\frac{l}{\sqrt{6}} \log_e (5 + 2\sqrt{6})$ . (10 + 10)
20. (a) Derive the moment of inertia of a thin uniform rod of length  $2a$  and mass  $M$  about a line through one end and perpendicular to it.
- (b) A circular disc of mass 30 kgms and radius 1 metre is mounted axially and rotates at the rate of 100 revolutions per minute. Find the kinetic energy of rotation. (10 + 10)

