STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011-12 & thereafter)

SUBJECT CODE : 11MT/MC/VL64

B. Sc. DEGREE EXAMINATION, APRIL 2017
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE : MAJOR CORE
PAPER : VECTOR SPACES AND LINEAR TRANSFORMATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL QUESTIONS. **(10X2=20)**

1. Define internal direct sum of a vector space.
2. Prove that kernel of a homomorphism is a subspace.
3. In $F^{(3)}$, the vector space of 3-tuples over F, the vectors $1, 1, 0$, $3, 1, 3$ and $(5, 3, 3)$ are linearly dependent.
4. Define a basis of a vectors space and give an example.
5. Prove that the inner product is conjugate symmetry in second variable over complex field.
6. Prove that orthogonal complement of a subspace of an inner product space is a subspace.
7. Define algebra and give an example.
8. In $A\ V$, algebra of linear transformations on V over F, is every right-invertible element is also left invertible? Justify.
9. Consider the linear transformation $T: R^3 \to R^2$, defined by $T\ x, y, z\ =\ x + y, 2z$. Find the matrix of T with respect to the bases $1, 1, 0$, $0, 1, 4$, $1, 2, 3$ and $1, 0$, $0, 2$.
10. When do we say a matrix is diagonalizable?

SECTION –B

ANSWER ANY FIVE QUESTIONS. **(5X8=40)**

11. If A and B are subspace of V prove that $(A + B)/B$ is isomorphic to $A/(A \cap B)$.
12. If $v_1, \dots v_n$ is a basis of V over F and if $w_1, \dots w_m$ in V are linearly independent over F, then prove that $m \leq n$.
13. State and prove Schwarz inequality.
14. Let V be the inner product space of polynomials, in a variable x, over the real field F of degree 2 or less with inner product defined by $p\ x\ , q(x)\ =\ {}_{-1}^{1} p\ x\ q\ x\ dx$. Find orthonormal basis corresponding to the basis $1, x, x^2$.
15. If $\lambda \in F$ is a characteristic root of $T \in A\ V$, then prove that λ is a root of the minimal polynomial of T. Also, prove that T has only a finite number of characteristic roots in F.

16. If V is finite dimensional over F, then prove that $T \in A(V)$ is regular if and only if T maps V onto V.

17. Examine whether the following matrix $\begin{pmatrix} 5 & -3 \\ 3 & -1 \end{pmatrix}$ is diagonalizable or not.

SECTION –C

ANSWER ANY TWO QUESTIONS. (2X20=40)

18. a) If V is a finite dimensional vector space and if W is a subspace of V, then prove that

    i) W is also finite dimensional

    ii) $\dim W \leq \dim V$

    iii) $\dim V/W = \dim V - \dim W$.

    b) If V *and* W are of dimensions m *and* n, respectively, over F, then prove that $Hom(V, W)$ is of dimension mn over F.

19. a) State and prove Gram-Schmidt orthgonalization process.

    b) If A is an algebra with unit element, over F, then prove that A is isomorphic to a subalgebra of A(V) for some vector space V over F.

20. a) If $T \in A(V)$ and if $dim_F V = n$, and if T has n distinct characteristic roots in F, then prove that there is a basis of V *over* F which consists of characteristic vectors of T.

    b) Consider the linear operator $T(x, y) = (2x, x + y)$ *on* R^2. Find the matrix of T with the respect to the standard basis $B = \{(1, 0), (0, 1)\}$ R^2. Use similarity transformation $A' = P^{-1}AP$ to determine the matrix A' with respect to the basis $B' = \{(-2, 3), (1, -1)\}$.