STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011–12 & thereafter)

B. Sc. DEGREE EXAMINATION, APRIL 2017
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE : MAJOR CORE
PAPER : GRAPH THEORY AND COMBINETORICS
TIME : 3 HOURS   MAX. MARKS : 100

SECTION-A

ANSWER ALL QUESTIONS: 10 X 2 = 20

1. Define a complete graph and give an example.

2. Prove that $\delta \leq \dfrac{2q}{p} \leq \Delta$.

3. If G is not connected then prove that $\overline{G}$ is connected.

4. Prove that a graph G with p points and $\delta \geq \dfrac{p-1}{2}$ is connected.

5. Define an eulerin graph and give an example.

6. Define centre of a graph.

7. True or false: Petersen graph is planar.

8. State the pigeonhole principle.

9. Define generalized permutation.

10. Write the ordinary generating function of the sequence $\langle 1,2,3,4,....\rangle$

SECTION-B

ANSWER ANY FIVE QUESTIONS: 5 X 8 = 40

11. a) Show that in any group of two or more people, there are always two with
        exactly the same number of friends inside the group.
    b) Show that in any graph the number of points of odd degree is even.   (4+4)

12. a) Prove that $\Gamma(G) = \Gamma(\overline{G})$.
    b) Show that isomorphism is an equivalence relation among graphs.   (4+4)

13. a) Prove that a graph G with p points and $\delta \geq \dfrac{p-1}{2}$ is connected.
     b) Prove that every connected graph has a spanning tree.   (4+4)

14. In any connected plane *(p,q)* graph $p \geq 3$ prove that $q \leq 3p - 6$.Hence show that K_5 is not planar.

15. If G is a connected graph having V,E and F as the set of vertices, edges and faces respectively then prove that $|V| - |E| + |F| = 2$.

16. Find the Number of (a) 2-digit even numbers, (b) 2-digit odd numbers, (c) 2-digit odd numbers with distinct digits, (d) 2-digit even numbers with distinct digits.

17. Find sequence corresponding to a) $(3 + x)^3$, b) $3x^3 + e^{2x}$ and b) $2x^2 (1 - x)^{-1}$.

SECTION-C

ANSWER ANY TWO QUESTIONS: **2 X 20 = 40**

18. a) Prove that a graph G with at least two points is bipartite, if and only if its cycles are of even length.

   b) Let G_1 be a (p_1, q_1) graph and G_2 be a (p_2, q_2) graph. Prove that

    (i) $G_1 + G_2$ is a $(p_1 + p_2, q_1 + q_2)$ graph

    (ii) $G_1 \times G_2$ is a $(p_1 p_2, q_1 p_2 + q_2 p_1)$ graph   (10 + 10)

19. a) Show that the following statements are equivalent
    (i) G is eulerian.
    (ii) Every point of G has even degree.
    (iii) The set of edges of G can be partitioned into cycles.
   b) State and prove Dirac theorem for Hamiltonian graphs.

                                                  (10 + 10)

20. a) Find the number of permutations of the 1 digit through 9 in which
    i) none of the blocks 23, 45and678 appears.
    ii) none of the blocks 34, 45and738 appears
   b) Prove the following formula for the Fibonacci numbers:

$$f(n) = C(n,0) + C(n-1,1) + C(n-2,2) + \ldots$$ (10+10)

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