STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011–12 & thereafter)

SUBJECT CODE : 11MT/MC/CA64

B. Sc. DEGREE EXAMINATION, APRIL 2017
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE : MAJOR CORE
PAPER : COMPLEX ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION-A

ANSWER ALL QUESTIONS: **10 X 2 = 20**

1. Express $w = z + \frac{1}{z}$ in the for $u\ \ x, y\ \ + i\ v(x, y)$.
2. Show that $f\ \ z\ \ = xy + iy$ is everywhere continuous but it is not analytic.
3. Under the transformation $w = iz + 1$ show that the half plane $x > 0$ maps onto the half plane $v > 1$.
4. Find the image of the line $y = 0$ under the mapping $= \ e^z$.
5. State Cauchy's theorem.
6. Evaluate $\ _{z\ =\frac{1}{2}} \frac{3z-1}{z^3-z} dz$.
7. Write the Maclaurin's series expansion for the function $f\ \ z\ \ = \ sinz$.
8. Find the zeros of $f\ \ z\ \ = \frac{z^3-1}{z^3+1}$.
9. Find the residue of $f\ \ z\ \ = \frac{z+1}{z^2-2z}$ at its poles.
10. State Rouche's theorem.

SECTION-B

ANSWER ANY FIVE QUESTIONS: **5 X 8 = 40**

11. For the function $f\ \ z\ \ = \begin{matrix} \frac{x^2y^5(x+iy)}{x^4+y^{10}} & if\ \ z\ \ \neq 0 \\ 0 & if\ \ z = 0 \end{matrix}$ prove that Cauchy Riemann equations are satisfied at $z = 0$ but not differentiable at $z = 0$.
12. Find the image of the strip $2 < x < 3$ under the transformation $= \frac{1}{z}$.
13. State and prove fundamental theorem of algebra.

14. Prove that the function $f(z)$ comes arbitrarily close to any complex number c in every neighbourhood of an essential singularity.
15. State and prove Cauchy's residue theorem.
16. Show that $u = log\sqrt{x^2 + y^2}$ is harmonic and determine its conjugate and hence find the corresponding analytic function.
17. Show that $\int_0^{2\pi} \frac{d\theta}{1+a\,sin\theta} = \frac{2\pi}{\sqrt{1-a^2}}$, $a^2 < 1$.

SECTION-C

ANSWER ANY TWO QUESTIONS: **2 X20 = 40**

18. a) Derive Cauchy-Riemann equations in polar form.
    b) Discuss the mapping $w = sinz$. (6 + 14)
19. a) State and prove Maximum modulus principle.
    b) Show that $\int_0^{2\pi} \frac{cos2\theta d\theta}{5+4\,cos\theta} = \frac{\pi}{6}$. (12 + 8)
20. a) State and prove Laurent's theorem.
    b) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent series for $1 < |z| < 3$. (14 + 6)

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