STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16)

SUBJECT CODE : 15MT/PC/TO34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2016
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : CORE
PAPER : TOPOLOGY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL THE QUESTIONS: (5 x 2 = 10)

1. Write down a collection of subsets of $X = \{1,2,3\}$ which is not a topology on X.
2. Define a locally connected topological space.
3. Define a Lebesgue number for an open covering of a metric space.
4. Define a second countable space.
5. Define a homeomorphism.

SECTION – B

ANSWER ANY FIVE QUESTIONS: (5 x 6 = 30)

6. Let X be a topological space. Suppose that $\mathbb{C}$is a collection of open sets of X such that for each open set U of X and each x in U, there is an element C of $\mathbb{C}$ such that $x \in C \subset U$, then show that $\mathbb{C}$ is a basis for the topology on X.
7. Prove that the image of a connected space under a continuous map is connected.
8. Prove that every closed subspace of a compact space is compact.
9. Prove that every metrizable space is normal.
10. State and prove Pasting lemma.
11. Let Y be a subspace of X and A be a subset of Y. If $\bar{A}$ denotes the closure of Ain X, then prove that the closure of A in Y equals $\bar{A} \cap Y$.
12. State and prove extreme value theorem.

SECTION – C

ANSWER ANY THREE QUESTIONS: (3 x 20 = 60)

13. a) Let $\mathbb{B}$ and $\mathbb{B}'$ be bases for the topologies τ and τ' respectively on X.Prove that the following are equivalent.

i) τ' is finer than τ.

ii) For each $x \in X$ each element $B \in \mathbb{B}$containing x, there is an $B' \in \mathbb{B}'$ suchthat $x \in \mathbb{B}' \subset \mathbb{B}$.

b) If Y is a subspace of$X$, then show that a set A is closed in Y if and only if itequalsthe intersection of a closed set of X with Y.

14. a) If L is a linear continuum in the order topology then prove that L is connected.

b) Prove that a space X is locally connected if and only if for every open set$U$of$X$, each component of$U$ is open is$X$.

15. Prove that the product of finitely many compact spaces is compact.

16. State and prove Urysohn'smetrization theorem.

17. a) If X and Y are topological spaces and if $f: X \to Y$ then show that the following are equivalent.

i) f is continuous.

ii) for every subset A of X, we have $f(\bar{A}) \subset \overline{f(A)}$.

iii) for every closed set $B$of$Y$ the set $f^{1}(B)$ is closed in X.

b) Let $\{X_\alpha\}$ be an indexed family of spaces, and let $A_\alpha \subset X_\alpha$ for each α.If$\pi X_\propto$is given either in the product or box topology, thenshow that $\Pi \bar{A}_\propto = \overline{\Pi A_\propto}$.

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