STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16& thereafter)

SUBJECT CODE : 15MT/PC/RA14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2016
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : REAL ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 2 = 10)**
ANSWER ALL QUESTIONS

1. Let x and y denote points in R^n.then prove that $\|x + y\| \leq \|x\| + \|y\|$.
2. Define a double series.
3. State Weierstrass M – Test.
4. Define the Directional Derivative of f at c in the direction u.
5. If $f = u + iv$ is a complex valued function with a derivative at a point $z$in C,then prove that, $J_f(z) = |f'(z)|^2$.

SECTION – B **(5 X 6 = 30)**
ANSWER ANY FIVE QUESTIONS

6. Prove that a set S in $R^n$is closed,if and only if,it contains all its adherent points.
7. Let S be a compact subset of R^n. Then prove that S is closed and bounded.
8. Show that if a series is convergent with sum S, then it is also $(C, 1)$ summable with cesaro sum S.
9. State and prove the Cauchy condition for the uniform convergence of a sequence of functions.
10. Prove by an example, that a function can have a finite directional derivative $f'(c; u)$for every$u$, but still may fail to be continuous at$c$.
11. State and prove the Mean-Value theorem for Differentiable functions.
12. Let A be an open subset of $R^n$and assume that $f: A \to R^n$ has continuous partial derivatives $D_j f_i$on A. If$J_f(x)$ is non-zero for all x in A, then prove that, f is an open mapping.

SECTION – C **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

13. State and prove the Bolzano – Weirestrass theorem on the bounded subsets of R^n.
14. State and prove the Merten's theorem on Cauchy product of two series.
15. Obtain the form of Taylor's formula in which the error term is expressed as an integral and hence prove the Bernstein's theorem.
16. If two partial derivatives $D_r f$ and $D_k f$ exists in an n–ball $B\ (\boldsymbol{c}: \delta)$ and if both are differentiable at c, prove that Dr $, k\ f(c)\ =\ Dk, r\ f(c)$.
17. State and prove the Implicit function theorem.

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