STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16& thereafter)

SUBJECT CODE : 15MT/PC/MA14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2016
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : MODERN ALGEBRA
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL THE QUESTIONS: **(5 x 2 = 10)**

1. Show that the conjugacy relation defined on a group is an equivalence relation.
2. Define a Euclidean ring.
3. If $f(x), g(x)$ are nonzero elements in $F[x]$ thenprove that$deg\ f(x) \leq deg\ f(x)\ g(x)$.
4. When do you say that a root of a polynomial is of multiplicity m ?
5. Define solvable group.

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5 x 6 = 30)**

6. If p is a prime number and $p/o(G)$ then prove that$G$ has an element of order p.
7. Suppose that G is the internal direct product of $N_1, \dots \dots \dots \dots \dots \dots, N_n$. Then prove that for$i \neq j,\ N_i \cap N_j = (e)$, and if $a \in N_i,\ b \in N_j$then $ab = ba$.
8. State and prove Fermat′s theorem.
9. State and prove the Division Algorithm.
10. Let $f(x) \in F[x]$ be of degree $n \geq 1$. Then prove that there is an extension E of F of degree at most $n!$ in which $f(x)$ has n roots.
11. For any $f(x), g(x) \in F[x]$ and any $\alpha \in F$, prove that

    (i) $\big(f(x) + g(x)\big)' = f'(x) + g'(x)$

(ii) $(\alpha\, f(x))' = \alpha\, f'(x)$

    (iii) $\big(f(x)g(x)\big)' = f'(x)g(x) + f(x)\, g'(x)$

12. If K is the field of complex numbers and F is the field of real numbers, compute $G(K, F)$ and what is the fixed field of $G(K, F)$.

SECTION – C

ANSWER ANY THREE QUESTIONS: **(3 x 20 = 60)**

13. State and prove Sylow′s theorem.

14. a) Prove that $J[i]$ is a Euclidean ring.

b) Find all the units in $J[i]$.

c) If $a + bi$ is not a unit of $J[i]$ prove that $a^2 + b^2 > 1$. (12+4+4)

15. a) State and prove Gauss′ Lemma.

b) State and prove The Eisentein Criterion.

c)Prove that the polynomial $1 + x + \cdots \ldots . + x^{p-1}$, where p is a prime number, is irreducible over the field of rational numbers. (6+8+6)

16. a) Prove that the element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of$F$.

b) If L is an algebraic extension of K and if K is an algebraic extension of F, then prove that$L$ is an algebraic extension of F. (12+8)

17. a) If F is of characteristic 0 and if a, b are algebraic over F, then prove that there exists an element $c \in F(a, b)$ such that $F(a, b) = F(c)$.

b) If K is a field and if $\sigma_1, \ldots \ldots, \sigma_n$are distinct automorphismsof K, then prove that it isimpossible to find elements $a_1, \ldots \ldots, a_n$, not all 0, in K such that $a_1\sigma_1(u) + a_2\sigma_2(u) + \ldots\ldots + a_n\sigma_n(u) = 0$ for all $u \in K$. (12 + 8)

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