

STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086  
(For candidates admitted during the academic year 2015 – 16)

SUBJECT CODE: 15MT/PC/GT34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2016  
BRANCH I - MATHEMATICS  
THIRD SEMESTER

COURSE : CORE

PAPER : GRAPH THEORY

TIME : 3 HOURS

MAX. MARKS : 100

SECTION – A

( 5 X 2 = 10 )

ANSWER ALL THE QUESTIONS

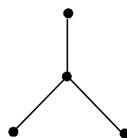
1. Define automorphism of a graph.
2. Define domination number of a graph.
3. Prove that in a critical graph, no vertex cut is a clique.
4. State Kuratowski's theorem.
5. Define dilation of the embedding.

SECTION – B

( 5 X 6 = 30 )

ANSWER ANY FIVE QUESTIONS

6. If  $G$  is a tree, Prove that  $\varepsilon = v - 1$ .
7. Let  $T$  be a spanning tree of a connected graph  $G$ , and let  $e$  be any edge of  $T$ . Prove that
  - (i) thecotree  $\bar{T}$  contains no bond of  $G$ ;
  - (ii)  $\bar{T} + e$  contains a unique bond of  $G$ .
8. If  $G$  is a  $k$ -regular bipartite graph with  $k > 0$ , Prove that  $G$  has a perfect matching.
9. Calculate the chromatic polynomial of the following graph.



10. If  $G$  is a connected plane graph, Prove that  $v - \varepsilon + \phi = 2$ .
11. Prove that a loopless digraph  $D$  has an independent set  $S$  such that each vertex of  $D$  not in  $S$  is reachable from a vertex in  $S$  by a directed path of length at most two.
12. State some fundamental properties of hypercube networks.

## SECTION – C

( 3 X 20 = 60 )

## ANSWER ANY THREE QUESTIONS

13. a) Prove that a graph is bipartite if and only if it contains no odd cycle.  
 b) Prove that a vertex  $v$  of a tree  $G$  is a cut vertex of  $G$  if and only if  $d(v) > 1$ .  
 (10 +10)
14. a) With usual notations, Prove that  $\kappa \leq \kappa' \leq \delta$ .  
 b) In a bipartite graph  $G$  with  $\delta > 0$ , Prove that the number of vertices in a maximum independent set is equal to the number of edges in a minimum edge covering.  
 (10 +10)
15. a) State and prove Brooks' theorem.  
 b) If  $G$  is a simple, Prove that either  $\chi' = \Delta$  or  $\chi' = \Delta + 1$ . (12 + 8)
16. a) State and prove the five colour theorem.  
 b) Prove that a digraph contains a directed path of length  $\chi - 1$ . (12 +8)
17. a) Explain any four basic principles of network design.  
 b) Draw de Bruijn digraph  $B(2, 3)$  and Kautz digraph  $K(2, 3)$ .  
 c) Define circulant digraph and state any four of its properties. (8 + 6 + 6)

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