STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600086 (For candidates admitted during the academic year 2015-16\& thereafter)

SUBJECT CODE : 15MT/PC/DE14

## M. Sc. DEGREE EXAMINATION, NOVEMBER 2016 <br> BRANCH I - MATHEMATICS <br> FIRST SEMESTER

| COURSE | CORE |
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| PAPER | : DIFFERENTIAL EQUATIONS |

TIME : 3 HOURS MAX. MARKS : 100

## SECTION - A

## ANSWER ALL QUESTIONS

1. Define Wronskian of two functions.
2. State Lipschitz condition.
3. Find a complete integral of $q=3 p^{2}$.
4. Define Boundary value problems.
5. Define Dirichlet problem.

## SECTION - B

$(5 \times 6=30)$

## ANSWER ANY FIVE QUESTIONS

6. If $P_{n}$ is a legendre polynomial then prove that $\int_{-1}^{1} P_{n}^{2}(t) d t=\frac{2}{2 n+1}$.
7. State and prove Abel's formula.
8. Find the complete integral of $\mathrm{zpq}=\mathrm{p}+\mathrm{q}$ by charpits method.
9. Reduce $\left(\frac{\partial^{2} z}{\partial x^{2}}\right)=x^{2}\left(\frac{\partial^{2} z}{\partial y^{2}}\right)$ to canonical form.
10. Derive one dimensional wave equation.
11. An insulated rod of lengthlhas its ends $A$ and $B$ at $0^{\circ} \mathrm{c}$ and $100^{\circ} \mathrm{c}$ respectively until steady state conditions prevail .If $B$ is suddenly reduced to $0^{\circ} \mathrm{c}$ and maintained at $0^{\circ} \mathrm{c}$ find the temperature at a distance $x$ from $A$ at time ' $t$ '.
12. Find the steady state temperature distribution in a thin rectangular plate bounded by the lines $x=0, x=a, y=0, y=b$. The edges $x=0, x=a, y=0$ are kept at temperature zero while the edge $y=b$ is kept at $100^{\circ} \mathrm{C}$.

## SECTION - C

$(\mathbf{3} \times 20=60)$

## ANSWER ANY THREE QUESTIONS

13. Solve the Bessel equation $t^{2} x^{\prime \prime}+t x^{\prime}+\left(t^{2}-p^{2}\right) x=0$ of order $p$.
14. State and prove picard's theorem.
15. (a)Find a complete integral of $z^{2}=p q x y$ by using charpits method.
(b)Reduce the equation $(n-1)^{2}\left(\frac{\partial^{2} z}{\partial x^{2}}\right)-y^{2 n}\left(\frac{\partial^{2} z}{\partial y^{2}}\right)=n y^{2 n-1}\left(\frac{\partial z}{\partial y}\right)$ to canonical form.
16. (a)Obtain the general solution of heat flow equation $k\left(\frac{\partial^{2} u}{\partial x^{2}}\right)=\frac{\partial u}{\partial t}$ by the method of separation of variables.
(b)Discuss D' Alembert's solution of wave equation.
17. Solve two dimensional Laplace equation in plane polar co-ordinates $(r, \theta)$.
