STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2015 – 16)

SUBJECT CODE: 15MT/PC/CA34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2016
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : CORE
PAPER : COMPLEX ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION–A

ANSWERALL QUESTIONS: **(5×2=10)**

1. Compute $\int_{|z|=2} \frac{dz}{z^2-1}$.
2. State the Mean value property on Harmonic functions.
3. Prove that $\zeta(s)\Gamma(s) = \int_0^\infty \frac{x^{s-1}}{e^x - 1} dx$.
4. Define Equicontinuous family and Normal family of functions.
5. When do we say $\phi(t)$ determines an analytic arc?

SECTION–B

ANSWERANYFIVEQUESTIONS: **(5×6=30)**

6. State and prove Cauchy's Theorem for a Rectangle.
7. If u_1 and u_2 are harmonic in a region Ω then prove that $\int_\gamma u_1 * du_2 - u_2 * du_1 = 0$.
8. For $\sigma = \text{Re}(s) > 1$, prove that $\zeta(s) = -\frac{\Gamma(1-s)}{2\pi i} \int_C \frac{(-z)^{s-1}}{e^z - 1} dz$, where $(-z)^{s-1}$ is defined on the complement of the positive real axis as $e^{(s-1)\log(-z)}$ with $-\pi < \text{Im}\log(-z) < \pi$.
9. Obtain a product representation of $\sin \pi z$.
10. Show that convergence with respect to ρ is equivalent to uniform convergence on all compact sets.
11. Prove that the functions $z = F(w)$ which map $|w|<1$ conformally onto polygons with angles $\alpha_k \pi (k = 1,2,.......n)$ are of the form $F(w) = C\int_0^w \prod_{k=1}^n (w - w_k)^{-\beta_k} dw + C'$, where $\beta_k = 1 - \alpha_k$, w_k are points on the unit circle and C and C' are complex constants.
12. Discuss the flow in the first quadrant $x > 0$, $y > 0$ whose complex potential is $\omega = z^2 = x^2 - y^2 + i2xy$, determining the stream line and stream function.

SECTION–C

ANSWERANYTHREEQUESTIONS: **(3×20 =60)**

13. a) State and prove Cauchy theorem on a disc.
    b) State and prove Cauchy Integral Formula.

14. a) Prove that if $f(z)$ is analytic in Ω then $\int_{\gamma} f(z)dz = 0$ for every cycle γ which is homologous to zero in Ω.
    b) Suppose that $u(z)$ is harmonic for $|z| < R$, continuous for $|z| \le R$, prove that $u(a) = \frac{1}{2\pi} \int_{|z|=R} \frac{R^2 - |a|^2}{|z-a|^2} u(z) d\theta$ for all $|a| < R$.

15. a)State and prove Mittag –Leffler theorem.
    b) Prove that $\varsigma(s) = 2^s \pi^{s-1} Sin \frac{\pi s}{2} \Gamma(1-s)\varsigma(1-s)$.Using this result derive the Legendre's duplication formula.

16. a) State and prove Arzela Ascoli theorem.
    b)Prove that a locally bounded family of analytic functions has locally bounded derivatives.

17. State and prove Riemann Mapping Theorem

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