STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted during the academic year 2015–16)

SUBJECT CODE: 15MT/MC/VA34

B. Sc. DEGREE EXAMINATION, NOVEMBER 2016
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : MAJOR – CORE
PAPER : VECTOR ANALYSIS AND ITS APPLICATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION-A
Answer All the questions **(10 x 2 = 20)**

1. Suppose $A = 5u^2\imath + uJ - u^3k$ and $B = \sin u\imath - \cos uJ$. Find $\frac{d(A.B)}{du}$.
2. Define continuity of scalar & vector function.
3. Define curl.
4. Suppose $A = x^2z^2\imath - 2y^2z^2J + xy^2zk$. Find $div\ A$ at the point $p(1,-1,1)$.
5. Define volume integrals.
6. If $f(t) = (3t^2 - t)\imath + (2 - 6t)J - 4tk$, find $\int_2^4 f(t)\,dt$.
7. Write Frenet-Serret formulae.
8. Define arc length.
9. Find the area of the ellipse $x = a\cos\theta\,, y = b\sin\theta$.
10. State Stokes' theorem.

SECTION-B
Answer any FIVE questions **(5 x 8 = 40)**

11. Suppose a particle P moves along a curve whose parametric equations, where $'t'$ is time, follows: $x = 40t^2 + 8t,\ y = 2\cos 3t\,,\ z = 2\sin 3t$.
    (a) Determine its velocity & acceleration at any time.
    (b) Find the magnitudes of the velocity and acceleration at $t = 0$.
12. (a) Suppose $\varphi(x,y,z) = xy^2z$ and $A = xz\imath - xy^2J + yz^2k$. Find $\frac{\partial^3(\varphi A)}{\partial x^2\partial z}$ at the point $(2,-1,1)$.
    (b) Show that $.\frac{dA}{dt} = A\frac{dA}{dt}$.
13. Show that
    (a) The vector $A = 3y^4z^2\imath + 4x^3z^2J - 3x^2y^2k$ is solenoidal and
    (b) The vector $B = (6xy + z^3)\imath + (3x^2 - z)J + (3xz^2 - y)k$ is irrotational.

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14. If the acceleration of a particle at any time t ≥ 0 is given by

$a = \frac{d^2r}{dt^2} = (25\cos 2t)\imath + (16\sin 2t)\jmath + (9t)k$ then fine the displacement.

15. Evaluate $\int_V \overrightarrow{Fdv}$, where $F = xy\imath - zx\jmath + k$, and V is the octant of the sphere

$x^2 + y^2 + z^2 = 4,\ x \geq 0,\ y \geq 0,\ z \geq 0.$

16. Determine a unit normal to the following surface, where $a > 9$,

$r = a\cos u \sin v\,\imath + a\,sin\,u \sin v\jmath + a\cos v\,k.$

17. Verify Green's theorem in the plane $\oint_C (xy + y^2)dx + x^2dy$ where C is the closed curve of the region bounded by $y = x$ & $y = x^2$.

SECTION-C
Answer any TWO questions **(2 x 20 = 40)**

18. Suppose $A = (3x^2 + 6y)\imath - 14yz\jmath + 20xz^2k.$ Evaluate $\int_c A\ dr$ from $(0,0,0)$ to $(1,1,1)$ along the following paths C.

(a) $x = t,\ y = t^2,\ z = t^3.$

(b) The straight lines from $(0,0,0)$ to $(1,0,0)$, then to $(1,1,0)$ and then to $(1,1,1)$.

(c) The straight line joining $(0,0,0)$ and $(1,1,1)$.

19. Express in cylindrical coordinates the quantities

(a) $\nabla\varphi$ (b) $\nabla.A$ (c) $\nabla \times A$ (d) $\nabla^2\varphi$

20. Verify Divergence theorem for $F = (x^2 - yz)\imath + (y^2 - zx)\jmath + (z^2 - xy)k$ taken over the rectangular parallelepiped $0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c$.