STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted during the academic year 2015-2016)

SUBJECT CODE : 15MT/MC/GT34

B. Sc. DEGREE EXAMINATION, NOVEMBER 2016
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : MAJOR – CORE
PAPER : INTRODUCTION TO GRAPH THEORY
TIME : 3 HOURS **MAX. MARKS : 100**

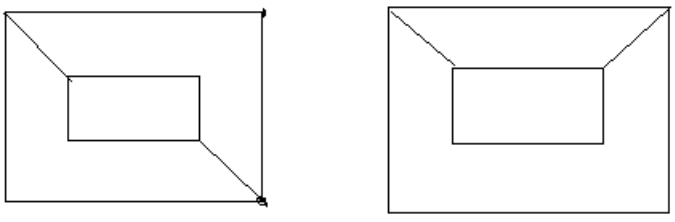
SECTION – A (10X2=20)
ANSWER ALL THE QUESTIONS

1. Define a bipartite graph graph and give an example.
2. Prove that $\delta \le \frac{2q}{p} \le \Delta$.
3. Define a walk in a graph.
4. Prove that a graph G with p points and $\delta \ge \frac{p-1}{2}$ is connected.
5. Show that every Hamiltonian graph is 2-connected.
6. Define closure of a graph.
7. True or false: Every sub graph of a planar graph is planar.
8. Write the crossing number of K_5.
9. Show that every nontrivial tree has atleast two vertices of degree 1.
10. Define centre of a tree.

SECTION – B (5X8=40)
ANSWER ANY FIVE QUESTIONS

11. a) Show that in any group of two or more people, there are always two with exactly the same number of friends inside the group.
    b) Prove that any self complementary graph has *4n* or *4n+1* points. (4 + 4)

12. Show that the following graphs are not isomorphic.



13. Prove that a graph G is connected if and only if any partition of V into subsets V_1 and V_2 there is a line of G joining a point of V_1 to a point of V_2.

14. If G is a graph with $p \geq 3$ vertices and $\delta \geq \frac{p}{2}$, then show that G is Hamiltonian.

15. If G is a connected graph having V, E and F as the set of vertices, edges and faces respectively then prove that $|V| - |E| + |F| = 2$

16. In any connected plane *(p,q)* graph without triangles and $p \geq 3$ then prove that $q \leq 2p - 4$.Hence show that $K_{3,3}$ is not planar.

17. Show that every tree has a centre consisting of either one point or two adjacent points.

SECTION – C **(2X20=40)**

ANSWER ANY TWO QUESTIONS

18. Prove the maximum number of lines among all p point graphs with no triangles is $\left[\frac{p^2}{4}\right]$

19. a) State and prove Chavatal theorem for hamiltonian graphs.

    b) Prove that the Petersen graph is nonhamiltonian.

(8 + 12)

20. Let G be graph. Then prove that following are equivalent.

    i) G is a tree.

    ii) Every two points of G are joined by a unique path.

    iii) G is connected and $p = q + 1$.

    iv) G is acyclic and $p = q + 1$.